Exceptional points and phase transitions in non-Hermitian binary systems

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Introduction

- A recent study [R. Hanai et al., Phys. Rev. Lett. 122, 185301, 2019] highlighted a first-order-like dissipative phase transition in a two-component quantum system with an exceptional point coinciding with phase boundary endpoint. Here we show a disparity between the exceptional point and the endpoint which is closely connected to the stability of solutions.
- We present a general phase diagram describing different phases in a generic nonlinear binary system. In a certain range of parameters, the system converges to a limit cycle, which vanishes at the exceptional point. Our results emphasize the connection between phase transitions, bistability, and exceptional points of non-Hermitian nonlinear systems in general, providing insight into strongly coupled light-matter systems in particular.
- We find that the model under investigation is incomplete unless nonlinear saturation of gain is taken into account. Importantly, saturation increases the complexity of the phase diagram and leads to the appearance of bistability.
- We find that while the first-order-like phase transition line with an endpoint is present, the equivalence of the endpoint to an exceptional point as found in [Hanai] is no longer valid in the general case. The phase diagram of [Hanai] can be restored in the limit of strong saturation

Model

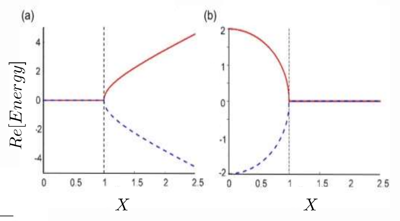
$$i\hbar\partial_t|\Psi\rangle = H|\Psi\rangle \text{ with } |\Psi\rangle = (\psi_C,\psi_X)^{\mathrm{T}}$$

$$H = \begin{pmatrix} E_C - i\hbar\gamma_C & \hbar\Omega_R \\ \hbar\Omega_R & E_X + g|\psi_X|^2 + ip \end{pmatrix}.$$

$$g = g_1 - ig_2$$

$$E = \frac{1}{2}\Big[E_c - \mathcal{E} + i(\mathcal{P} - \hbar\gamma_c) \pm \sqrt{4\hbar^2\Omega_R^2 + [\mathcal{E} - E_c + i(\mathcal{P} + \hbar\gamma_c)]^2}\Big],$$

where $\mathcal{P} = p - g_2(n_X^{\mathrm{SS}})^2$ and $\mathcal{E} = E_x + g_1(n_X^{\mathrm{SS}})^2$.



the following conditions for the EP

$$p^{\mathrm{EP}} = \hbar\Omega_R + \frac{g_2\delta}{g_1}, \quad \gamma_C = \Omega_R. \tag{6}$$

This can occur when $n_X^{\mathrm{SS}} = \sqrt{\delta/g_1}$, that is, whenever the system is blue-detuned ($\delta > 0$). On the other hand,

EP and ET: A Separation

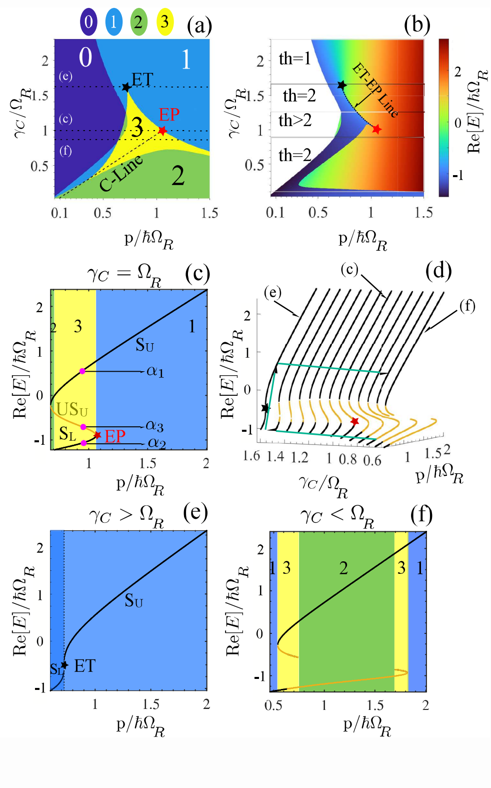


Fig. 1. (a-b): Phase diagrams of binary system (equations in the model section). In (a) the number of stationary states is marked with colors in the function of photon decay rate γ_C and pumping strength p. In (b) only the lowest-energy stable state is shown. Here colors indicate the real part of the energy. In (a) and (b) the exceptional point (EP, red star) and the endpoint of the first-order-like phase transition (ET) are shown. At the C-Line two solutions coalesce and periodic solution vanishes. Cross-sections of constant with different numbers of thresholds (th) are marked with horizontal lines. In (c) we show the case $\gamma_C=\Omega_R$, for which the energy eigenvalues coalesce at the EP , which is also a turning point of a bistability curve. A stable solution (marked by S and colored in black) may be obtained by increasing pumping. Stable solutions are marked with S and black lines, while unstable solutions are marked with US and orange lines. Panel (d) shows real part of energy for different pumping and decay rates. The ET point corresponds to the transition to bistability at $\gamma_C>\Omega_R$. This cross-section is depicted in panel (e), while in panel (f) we show the case $\gamma_C<\Omega_R$ where the unstable solution is split into two branches, and the lowest-energy solution becomes unstable.

Using parameters: δ=0.2 $\hbar\Omega_R$, g_1=0.1 $\hbar\Omega_R$, E_X=0, E_C=0.2 $\hbar\Omega$ and g_2=0.3 g_1.

Stability Analysis and Periodic Solutions

In order to perform the stability analysis we first rewrite the equations of motion for fields $\partial_t\psi_C$ and $\partial_t\psi_X$ in terms of densities $n_{X,C}$ and relative phase φ_{CX}. It yields $\partial_t n_C = F_1(n_X, n_C, \varphi_{CX})$, $\partial_t n_X = F_2(n_X, n_C, \varphi_{CX})$ and $\partial_t\varphi_{CX} = F_3(n_X, n_C, \varphi_{CX})$ where

$$F_1 = -\gamma_C n_C - \Omega_R n_X \sin\varphi_{CX}, \tag{1a}$$

$$F_2 = \frac{p}{\hbar}n_X - \frac{g_2}{\hbar}n_X^3 + \Omega_R n_C \sin\varphi_{CX}, \tag{1b}$$

$$F_3 = -\frac{\delta}{\hbar} + \frac{g_1}{\hbar}n_X^2 - \Omega_R\Big(\frac{n_X}{n_C} - \frac{n_C}{n_X}\Big)\cos\varphi_{CX}. \tag{1c}$$

The Jacobian matrix J is

$$J(n_C^{SS}, n_X^{SS}, \varphi_{CX}^{SS}) = \begin{pmatrix} \partial F_1/\partial n_C & \partial F_1/\partial n_X & \partial F_1/\partial\varphi_{CX} \\ \partial F_2/\partial n_C & \partial F_2/\partial n_X & \partial F_2/\partial\varphi_{CX} \\ \partial F_3/\partial n_C & \partial F_3/\partial n_X & \partial F_3/\partial\varphi_{CX} \end{pmatrix}_{(n_C^{SS}, n_X^{SS}, \varphi_{CX}^{SS})}. \tag{2}$$

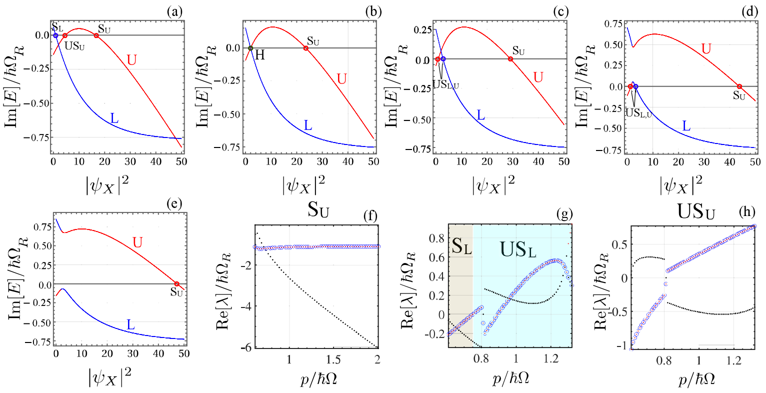
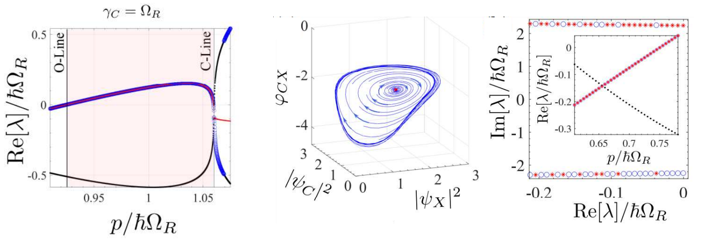


Fig. 5. Determination of the number of steady state solutions. Imaginary part of eigenvalue energy (Im[E]) versus exciton density $|\psi_X|^2$ is shown. Steady state solutions correspond to Im[E]=0. Stable and unstable solutions are labeled as S and US, respectively.



Phase Diagram at Large g_2

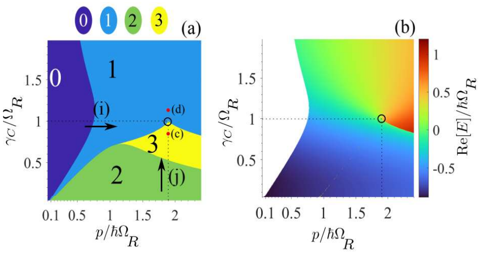


Fig. 6. Example of phase diagrams at large g_2; the exceptional point and the endpoint of phase transition are marked by a black open circle in (a) and (b) and they correspond to the same point in the phase diagram. Increasing pumping along path (i), there is only one solution at either side of the exceptional point. Increasing γ_C along the path (j), three solutions can coalesce at the exceptional point.

Conclusions

- We showed that, contrary to previous understanding, non-Hermitian two-mode systems exhibit first-order-like dissipative phase transition with an endpoint that in general does not coincide with the exceptional point.
- While the endpoint is where the bistability appears, the exceptional point is where the stable and unstable solutions coalesce. We demonstrated that first-order-like phase transition may occur in the weak coupling regime, and that for certain values of parameters one can predict oscillatory solutions, which converge to a stable exceptional point.
- We found a regime of limit cycle solutions due to a Hopf bifurcation, which eventually disappear at an exceptional point.

References

[1] R. Hanai ,et al., Phys. Rev. Lett. **122**, 185301 (2019).
[2] H. M. Price , et al., Phys. Rev. A **95**, 023607 (2017)
[3] T. Ozawa, et al., Nat. Rev. Phys. **1** 349 (2019)
[4] Y. Ashida et al., Adv. Phys., **69**, 249 (2020)
[5] M. -A. Miri et al., Science **363**, eaar7709 (2019)
[6] A. Rahmani et al., Phys. Rev. B **109**, 085311 (2024)

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