Correlations of strongly interacting ultra-cold p-wave fermions in one-dimensional trap

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P. Kościk, T. Sowiński: Phys. Rev. Lett. 130, 253401 (2023)

abstract

Ground-state properties of polarized fermions interacting via zero-range p-wave forces in a one-dimensional geometry are studied

We rigorously prove that in the limit of infinite attractions spectral properties of any-order reduced density matrix describing arbitrary subsystem are completely independent of the shape of an external potential

Quantum correlations between any two subsystems are in this limit insensitive to the confinement

the system

$$\mathcal{H}=\sum_{i=1}^{N}\left[-\frac{1}{2}\frac{\partial^2}{\partial x_i^2}+V(x_i)+\sum_{j=i+1}^{N}U(x_i-x_j)\right]$$

arbitrary confinement

p-wave interactions

$$U(x)=-\frac{g_F}{2}\overleftarrow{\frac{\partial}{\partial x}}\delta(x)\overrightarrow{\frac{\partial}{\partial x}}$$

We focus on the many-body ground state of this Hamiltonian

$$\Psi(x_1,..,x_N) \qquad [\mathcal{H}-\mathcal{E}_0]\,\Psi(x_1,..,x_N)=0$$

energy of the many-body ground state of the system

the Girardeau mapping

Regardless of the shape of the external potential, there is a rigorous ONE-TO-ONE mapping between the ground states of p-wave fermions and s-wave bosons

M. Girardeau: J. Math. Phys. (N.Y.) 1, 516 (1960)
T. Cheon, T. Shigehara: Phys. Rev. Lett. 82, 2536 (1999)

IF

we consider the corresponding system of interacting bosons

$$\mathcal{H}=\sum_{i=1}^{N}\left[-\frac{1}{2}\frac{\partial^2}{\partial x_i^2}+V(x_i)+g_B\sum_{j=i+1}^{N}\delta(x_i-x_j)\right]$$

with

$$g_B=-2/g_F$$

THEN

$$\Psi(x_1,..,x_N)=\Pi_{i<j}\mathrm{sgn}(x_i-x_j)\Psi_B(x_1,..,x_N)$$

limit of infinite attractions

In the limit of infinite attractions, $g_F \to -\infty$, the many-body ground state is known rigorously

$$\Psi(x_1,..,x_N)=\prod_{i=1}^{N}\left(\phi(x_i)\prod_{j=i+1}^{N}\mathrm{sgn}(x_i-x_j)\right)$$

mapping to the system of non-interacting bosons!

ground-state solution of the single-particle problem

$$\left[-\frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{d}x^2}+V(x)\right]\phi(x)=E_0\phi(x)$$

Natural observation

in principle, all ground-state properties should depend on the shape of external confinement

Internal correlations

The most general object encoding internal correlations in any many-body system of indistinguishable particles is the whole set of p-particle reduced density matrices

$$\rho^{(p)}(\boldsymbol{x}_p,\boldsymbol{x}'_p)=\int \mathrm{d}\boldsymbol{q}_p\Psi^*(\boldsymbol{x}_p,\boldsymbol{q}_p)\Psi(\boldsymbol{x}'_p,\boldsymbol{q}_p)$$

$$\boldsymbol{x}_p=(x_1,\dots,x_p)$$

$$\boldsymbol{q}_p=(x_{p+1},\dots,x_N)$$

One can perform its spectral decomposition

$$\int \mathrm{d}\boldsymbol{x}'_p\,\rho^{(p)}(\boldsymbol{x}_p,\boldsymbol{x}'_p)u_k(\boldsymbol{x}'_p)=\lambda_k^{(p)}u_k(\boldsymbol{x}_p)$$

and write in the form

$$\rho^{(p)}(\boldsymbol{x}_p,\boldsymbol{x}'_p)=\sum_k\lambda_k^{(p)}u_k(\boldsymbol{x}_p)u_k^*(\boldsymbol{x}'_p)$$

Internal correlations

Eigenvalues of the reduced density matrix may be used to quantify correlations between subsystems

1. von Neumann entropy

$$\mathcal{S}[\hat\rho^{(p)}]=-\mathrm{Tr}\left[\hat\rho^{(p)}\log\hat\rho^{(p)}\right]=-\sum_k\lambda_k^{(p)}\log\lambda_k^{(p)}$$

2. purity

$$\mathcal{P}[\hat\rho^{(p)}]=\mathrm{Tr}\left[\left(\hat\rho^{(p)}\right)^2\right]=\sum_k\left(\lambda_k^{(p)}\right)^2$$

3. participation number

$$\mathcal{K}[\hat\rho^{(p)}]=\frac{1}{\mathrm{Tr}\left[\left(\hat\rho^{(p)}\right)^2\right]}=\frac{1}{\sum_k\left(\lambda_k^{(p)}\right)^2}$$

the rigorous result

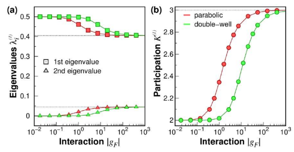
in the limit of infinite interactions eigenvalues of any reduced density matrix describing fermionic ground state ARE INDEPENDENT on the shape of the trap

The simplest example (numerical visualisation)

$$N=2$$

$$V_1(x)=\frac{x^2}{2}$$

$$V_2(x)=\frac{x^2}{2}+4\mathrm{e}^{-x^2/2}$$



Numerical diagonalization for different interaction strengths

$$\Psi(x_1,x_2)\longrightarrow\rho^{(1)}(x_1,x'_1)=\int \mathrm{d}x_2\,\Psi^*(x_1,x_2)\Psi(x'_1,x_2)$$

sketch of the proof

many-body ground state

$$\Psi(x_1,..,x_N)=\prod_{i=1}^{N}\left(\phi(x_i)\prod_{j=i+1}^{N}\mathrm{sgn}(x_i-x_j)\right)$$

single-particle reduced density matrix

$$\rho^{(1)}(x,x')=\phi(x)\phi(x')\left(\int \mathrm{d}z\,\phi^2(z)\,\mathrm{sgn}(x-z)\mathrm{sgn}(x'-z)\right)^{N-1}$$

ϕ-dependent eigenequation

$$\int \mathrm{d}x'\,\rho^{(1)}(x,x')u_k(x')=\lambda_k^{(1)}u_k(x)$$

cumulative distribution

$$F(x)=\int_{-\infty}^{x}\mathrm{d}z\,\phi^2(z)$$

change of variables

$$\xi=F(z) \qquad y=F(x)$$

$$\mathrm{d}\xi=\phi^2(z)\mathrm{d}z \qquad y'=F(x')$$

$$u_k(x)=\phi(x)v_k(F(x))$$

another ϕ-independent eigenequation

$$\int_0^1\mathrm{d}y'\,\rho_0^{(1)}(y,y')v_k(y')=\lambda_k^{(1)}v_k(y)$$

$$\rho_0^{(1)}(y,y')=(1-2|y-y'|)^{N-1}$$

The reasoning for higher reduced density matrices is the same

References

PHYSICAL REVIEW LETTERS **130,** 253401 (2023)

Universality of Internal Correlations of Strongly Interacting *p*-Wave Fermions in One-Dimensional Geometry

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(Received 10 March 2023; revised 27 April 2023; accepted 5 June 2023; published 23 June 2023)

We consider the many-body ground state of polarized fermions interacting via zero-range *p*-wave forces in a one-dimensional geometry. We rigorously prove that in the limit of infinite attractions spectral properties of any-order reduced density matrix describing arbitrary subsystem are completely independent of the shape of an external potential. It means that quantum correlations between any two subsystems are in this limit insensitive to the confinement. In addition, we show that the purity of these matrices quantifying the amount of quantum correlations can be obtained analytically for any number of particles without diagonalizing them. This observation may serve as a rigorous benchmark for other models and methods describing strongly interacting *p*-wave fermions.

DOI: 10.1103/PhysRevLett.130.253401

Ministry of Education and Science
Republic of Poland

2022/WK/06

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2023/49/B/ST2/03744