Spectral correlation functions in the studies of quantum and wave systems

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Summary of Scientific Achievements

Appendix No. 3

Contents

1. Scientific degrees and experience

Education and degrees:

- **PhD in Physics** (May 2009)
  Faculty of Physics, Warsaw University of Technology, Poland
  Title of the thesis: *Investigation of ultrafast LT GaAs photodetectors resolution.*
  carried out in the Laboratory of Femtosecond Techniques
  Supervisor - prof. nzw. dr hab. Bronisław Pura

- **MSc in Physics** (June 1993)
  Faculty of Mathematics, Physics and Chemistry, Field of study: Physics
  Nicolaus Copernicus University in Toruń, Poland
  Title of the thesis: *Phenomenon of opto - magnetic double resonance in* ^{48}*Cd atoms.*
  carried out in the Laboratory of Atomic Physics
  Supervisor - prof. nzw. dr hab. Franciszek Bylicki
  Scholarship of the Rector of UMK

- High School: II Liceum Ogólnokształcące, Olsztyn, Poland
  belonged to the Association of Active/Creative High Schools
  extended mathematics and physics

Information on work in research institutes:

- 09/2013 - currently Institute of Physics, Polish Academy of Sciences, Warsaw
  Division of Radiation Physics and Spectroscopy- specialist

- 06/2009 - 02/2011 Faculty of Physics, Warsaw University of Technology, Poland,
  research continuation in the Laboratory of Femtosecond Techniques

- 02/2006 - 09/2008 exercises for Physics lectures with students at the WUT

- 10/2001- 09/2005 PhD student at the Faculty of Physics, Warsaw University of Technology, exercises and laboratories with students at the WUT

2. Presentation of the achievements set out in the art. 219 para 1 point 2 of the Act of Law on higher education and science

2.1 Title and cycle of publications constituting the scientific achievement.

Spectral correlation functions in the studies of quantum and wave systems.

[H1] **M. Białous**, V. Yunko, Sz. Bauch, M. Ławniczak, B. Dietz, L. Sirko
Power spectrum analysis and missing level statistics of microwave graphs with violated time reversal invariance,
Physical Review Letters **117**, 144101 (2016)

[H2] B. Dietz, V. Yunko, **M. Białous**, S. Bauch, M, Ławniczak, L. Sirko
Nonuniversality in the spectral properties of time-reversal-invariant microwave networks and quantum graphs
Physical Review E **95,** 052202 (2017)

[H3] **M. Białous**, B. Dietz, L. Sirko
How time-reversal - invariance violation leads to enhanced backscattering with increasing openness of a wave-chaotic system,
Physical Review E **102**, 042206 (2020)

[H4] **M. Białous**, B. Dietz, L. Sirko
Missing level statistics in a dissipative microwave resonator with partially violated time-reversal invariance,
Physical Review E **103,** 052204 (2021)

[H5] **M. Białous**, L. Sirko
Enhancement factor in the regime of semi-Poisson statistics in a singular microwave cavity,
Physical Review E **106**, 064208 (2022)

[H6] **M. Białous**, B. Dietz, L. Sirko
Experimental study of the elastic enhancement factor in a three-dimensional wave-chaotic microwave resonator exhibiting strongly overlapping resonances,
Physical Review E **107**, 054210 (2023)

[H7] V. Yunko, **M. Białous,** L. Sirko
Edge switch transformation in microwave networks,
Physical Review E **102**, 012210 (2020)

2.2 Introduction

Statistical analysis of energy spectra play an important role in the nuclear and atomic physics. The degree of level correlation is an explicit indicator of systems chaoticity which exhibits features of quantum chaos [1-2]. Level fluctuations in quantum systems with chaotic classical analogue follow predictions of random matrix theory (RMT) [3-5]. Originally the RMT was devised for the analysis of energy spectra of heavy nuclei and afterwards applied in many areas of physics and mathematics [6-8].

According to the RMT assumption chaotic systems can be modeled by the three Gaussian ensembles, depending on symmetry of Hamiltonians $\mathcal{H}$: orthogonal (GOE), unitary (GUE) and symplectic (GSE) with half-integer spin $\pm 1/2$ [9-11]. Specified classes are labeled by Dyson index $\beta \in \{1,2,4\}$ that measures repulsion between consecutive energy levels. Numerical simulations and theoretical studies of the RMT corroborate that level spectral statistics determine dynamics of quantum system. In the opposite limit, the classically integrable system with regular dynamics and no correlation obeys Poisson statistics (β=0) [12-13]. It is well known that numerous of real physical systems display an intermediate dynamics [14-15], e.g. statistics of deformed nuclei are closer to Poisson while spherical ones reveal GOE distribution. Generally, if the nuclear mass increases, the deviation from GOE to Poisson is observed [16].

Standard RMT requires sufficiently long and complete sequences of energy levels with only one symmetry. Experimental spectra of real physical systems can't fulfill this condition due to absorption, missing levels or mixed symmetries [17-18]. Imperfect or incomplete spectra lead to ambiguous information about quantum system. A significant influence on spectra is also observed due to local transformations of system, e.g. "switch" operation [19-20] or the impact of Neumann and Dirichlet boundary conditions [5]. Thus, the proper spectral analysis are indispensable in order to extract essential information about levels interaction and symmetry class of system.

To investigate spectral fluctuations, the short- and long-range correlation functions of scattering matrices are applied [21-25]. They determine unambiguously the degree of chaoticity vs regularity. Level correlations depend on the fraction of missing levels in spectra. In the regime of strong overlapping or presence of many scattering channels, the elastic enhancement factor becomes an alternative and universal measure [26-29].

Due to some limitations the level dynamics in Gaussian β- ensemble spectra had not been examined properly so far. Above all, simulations of quantum system desire challenging measurements and advanced analysis of the spectroscopic spectra. The number of experimental groups involved in this area is very little around the world, therefore each experimental investigation of spectral properties of quantum systems within the framework of RMT is very relevant and precious.

The essential goal of my research refers to spectral fluctuations of experimentally simulated quantum systems, attributed to the fundamental symmetry classes in the RMT model, as well as an integrable system with regular dynamics. Obtained results expand current knowledge and reveal that spectral correlation functions play an important role in the studies of quantum and classical systems properties, even in particularly difficult cases when information about them is incomplete. Differentiated degree of energy levels correlation allows to determine their chaoticity and symmetry class. My contribution to the development of discussed research area is presented in the series of publications **[H1-H7]**.

My another scientific achievement **[D1-D3]** is associated with the research at the Faculty of Physics, Warsaw University of Technology. It included the construction of modern Laboratory of Femtosecond Techniques with an advanced and original Electro-Optic-Sampling system (EOS) for time-resolved experiments in THz domain and afterwards the investigation of ultrafast *LT* GaAs photodetectors of the newest generation. The realized research was of significant importance because of the wide applications of detectors: in military, astrophysics, medicine, telecommunication.

2.3 Description of scientific results [H1- H7]

Symmetry classes of quantum systems:

integrable	**semi-integrable**	**chaotic in the RMT (Wigner-Dyson statistics)**		
Poisson ⟷	**semi-Poisson** ⟷	**GOE** ⟷	**GUE** ⟷	**GSE**
$\beta=0$		$\beta=1$	$\beta=2$	$\beta=4$
		$\mathcal{T}$- preserved	$\mathcal{T}$- violated	$\mathcal{T}$- preserved
		$s=\pm 1$	$\vec{B}\neq 0$	$s=\pm 1/2$
quasi-degeneracy **no correlation**			**level repulsion** **correlation**	
	$0<\varphi<1$			
$\eta=1$ ⟶ $\eta=2$		$\xi=0$ ⟶ $\xi=1$		

The major part of information about dynamics of quantum systems, described mainly by non-relativistic Schrödinger equation, comes from experiments of the scattering microwave radiation [30-32]. An investigation of classical chaos as the quantum manifestation, may be realized using following physical systems [33-35]:

1) ***Microwave network that simulates quantum graph - system with one degree of freedom***

   Such simulation is possible because of the mathematical equivalency between the telegraph's equation $\nabla^2 U_{ij}(x) + \frac{\omega^2 \epsilon}{c^2} U_{ij}(x) = 0$ for signal propagation in a microwave network and the one-dimensional Schrödinger equation $-\nabla^2 \Psi_{ij}(x) = k^2 \Psi_{ij}(x)$ that describes the motion of particle in quantum graph [36-37]. This equivalency is performed only below a cut-off frequency $\nu_{max} = c/[\pi\sqrt{\varepsilon}(R_1+R_2)]$, (where c is the velocity of light in vacuum, R_1 and R_2 are the radius of cross-section of coaxial cable, ε is dielectric constant). Quantum graph is a metric graph $\Gamma = (\mathcal{V}, \mathcal{E})$ formed by the vertices $v \in \mathcal{V}$ and connected by the edges $e \in \mathcal{E}$. The physical properties of microwave network depend on its topology, the lengths of bonds and the vertex boundary conditions. The most common are the Dirichlet and Neumann boundary conditions.

2) ***Microwave resonator that simulates quantum billiard - system of two degrees of freedom***

   The stationary Helmholtz equation $\nabla^2 \vec{E} = -k^2 \vec{E}$ with Dirichlet boundary conditions at the side walls of the resonator describes the electromagnetic field, where $k = \frac{2\pi\nu}{c}$ (ν denotes the frequency). Whereas, two-dimensional Schrödinger equation $\nabla^2 \Psi = -k^2 \Psi$ for the wave function $\Psi_{ij}(x)$, with $k = \sqrt{\frac{2mE}{\hbar^2}}$ (m and E denote the mass and energy of particle) describes quantum billiard of the corresponding shape [38-39]. These two equations are mathematically equivalent for the resonator of height h only below the cut-off frequency $\nu_{max} = c/(2h)$ when the transversal - magnetic TM_{00} mode can propagate. The chaoticity degree of classical dynamics depends only on billiard's shape.

In the high precision measurements of the two port scattering matrix $\hat{S}(\nu)$, reflections S_{11}, S_{22} and transmissions S_{12}, S_{21} are obtained. Matrix elements are measured using Agilent E8364B PNA network analyzer (VNA) with two flexible microwave cables which perform the role of leads. The number of spectra resonances is predicted by the Weyl's formula [40]. To investigate level correlations which determine the degree of systems chaoticity in the quantum domain, I used different statistical measures depending on systems properties.

Quantum system with preserved and violated $\mathcal{T}$- invariance [H1-H2]

An imperfect experimental spectra typical for the real physical systems have an impact on the energy level correlation, leading to incomplete information about system. This problem has been examined experimentally and numerically only for systems with preserved time reversal ($\mathcal{T}$) invariance [41-45]. Whereas, the experimental study of fluctuations in incomplete spectra for the system with broken time-reversal symmetry (TIV) was presented for the first time in our publication **[H1]**. Spectral properties of system with Dyson index β=2 are well described by those of random matrices from the Gaussian unitary ensemble (GUE). I explored effect of symmetry breaking on levels statistics. The measure of average power spectrum $S(\tilde{k})$ was incorporated in order to describe quantum system more precisely.

To simulate quantum graph with TIV we used microwave network of the constant optical length $\mathcal{L}$=7.2 m composed of six junctions, connected each other by coaxial cables SMA-RG402 (Fig.1(a)). The symmetry violation was realized by five microwave circulators operated in the frequency window 7-14 GHz (the cut-off frequency ν=33.26 GHz). The element $S_{11}(\nu)$ of the scattering matrix was measured for 30 graphs realizations, by changing length of phase shifter in steps of ± 1.12 mm. In the analysis of correlation functions I have taken 7500 identified eigenvalues ν_i with the uniform levels density in the 1 GHz window.

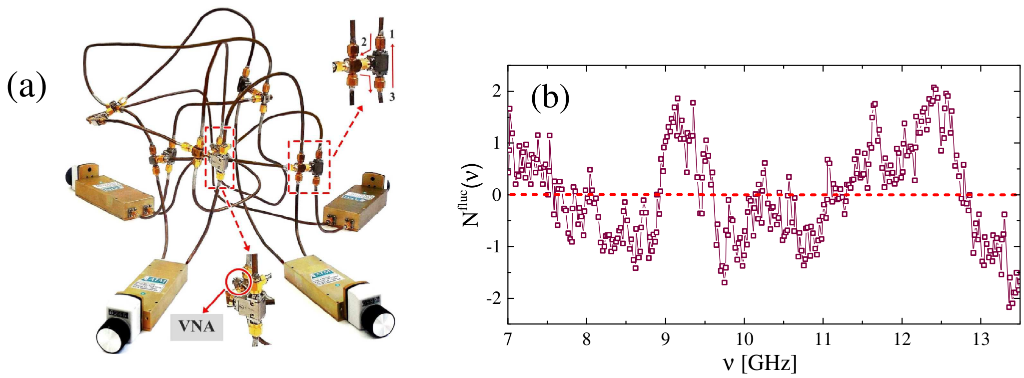


Fig. 1 ***(a)*** *A six vertex microwave network with five circulators* Anritsu PE8403 *and four phase shifters represents GUE system. Network was coupled to the microwave analyzer VNA (Agilent E8364B) for the scattering matrix measurement. Waves in circulator propagate through ports 1→2, 2→3 and 3→1. Photograph in* ***[H1]****.* ***(b)*** *Example of fluctuating part* $N^{fluc}(\nu)=N(\nu) - N^{Weyl}(\nu)$.

Before examining the spectral properties of system, the ordered resonance frequencies $\nu_i \leq \nu_{i+1}$ are unfolded by employing Weyl's law $\epsilon_i = N^{Weyl}(\nu_i)$. The cumulative spectral density is expressed by sum $N(\nu_i) = N^{Weyl}(\nu_i) + N^{fluc}(\nu_i)$, whereas only fluctuating part is associated to chaotic or integrated properties. Unfolding procedure remove specific properties of system, leading to dimensionless eigenvalues ϵ_i with the average spacing unity $< s >= 1$ between adjacent levels, while $s_i = \epsilon_{i+I} - \epsilon_i$. [30]. I controlled the jumps by more than 1 in the fluctuating part $N^{fluc}(\nu_i)$ to check missing or spurious eigenfrequency (Fig.1((b)).

To explore system I applied the standard measure of spectral regularity, that is, the nearest-neighbor spacing distribution P(s) (NNSD) [46]. It behaves as $P(s) \sim s^{\beta}$ (when $s \ll 1$) and measures the short-range spectral fluctuations; it indicates the degree of levels repulsion. Including the fraction of identified eigenfrequencies φ, with the assumption that some levels 1− φ are missing randomly and 0<φ< 1, the NNSD is approximated by

$$p(s) \cong P\left(\frac{s}{\varphi}\right) + (1-\varphi)P\left(1, \frac{s}{\varphi}\right) + \cdots \qquad (1)$$

Figures 2 (a-b) illustrate agreement between experimental results (histogram and points) and analytical curves (red dashed lines) for P(s) and its integral I(s) for $\varphi = 0.965 \pm 0.005$. Solid black line represents GUE in the RMT framework. I also examined the long-range correlation functions in terms of the number variance $\Sigma^2(L)$ of eigenvalues in an interval of the length L and the spectral rigidity $\Delta_3(L)$ that is the mean-square deviation of the integrated resonance density from the fit. Analytical curves for the long interactions also take into account the fraction of identified resonances φ

$$\sigma^2(L) = (1-\varphi)L + \varphi^2\Sigma^2\left(\frac{L}{\varphi}\right) \qquad (2)$$

$$\delta_3(L) = (1-\varphi)\frac{L}{15} + \varphi^2\Delta_3\left(\frac{L}{\varphi}\right) \qquad (3)$$

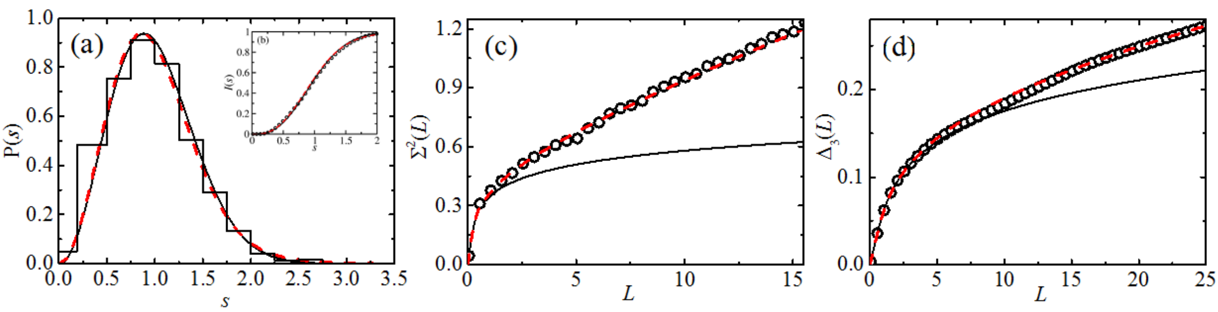


Fig. 2 *Experimental results averaged for 30 realizations of networks, each of them contains 250 identified resonance frequencies.* ***(a)*** *the NNSD,* ***(b)*** *its integral I(s),* ***(c)*** *the number variance* Σ^2 *and* ***(d)*** *the rigidity* Δ_3. *Results are compared to the RMT (black solid lines) and the corresponding missing-level statistics for* $\varphi = 0.965$ *(red dashed lines). Results in* ***[H1].***

The NNS distribution involves only two-levels interactions between nearest-neighbors. Whereas long–range spectral fluctuations include also interactions between further levels, therefore the deviations from RMT model are observed in statistical distributions more distinctly (Figs.2 (c-d)). In order to verify discussed behavior a new measure of the chaoticity degree was additionally introduced, that is an average power spectrum $S(\tilde{k})$ [23-25]. It measures long-range correlation and is expressed as the discrete Fourier transform

$$S(\tilde{k}) = |\tilde{\delta}_q|^2 = \left| \frac{1}{\sqrt{N}} \sum_{q=0}^{N-1} \delta_q \exp\left(-2\pi\, i \tilde{k}\, q\right) \right|^2 \tag{4}$$

for unfolded N levels of energy ϵ_i, where $\delta_q = \epsilon_{q+1} - \epsilon_1 - q$ means the deviation of the q-th nearest- neighbor spacing from its mean value q. This measure displays dependence $\langle S(\tilde{k})\rangle \propto \tilde{k}^{-\alpha}$, where α=2 for regular system and α=1 for chaotic one [45]. The analytical expression for the average power spectrum including incomplete spectra is given by formula

$$\langle s(\tilde{k})\rangle = \frac{\varphi}{4\pi^2}\left[\frac{K(\varphi\tilde{k})-1}{\tilde{k}^2} + \frac{K[\varphi(1-\tilde{k})]-1}{(1-\tilde{k})^2}\right] + \frac{1}{4\sin^2(\pi\tilde{k})} - \frac{\varphi^2}{12} \tag{5}$$

where $0 \leq \tilde{k} \leq 1$. Experimental results for $\varphi = 0.965 \pm 0.005$ below $\log_{10}(\tilde{k}) \leq -0.5$ start to deviate from GUE (red dash line in Fig.3(a)). The value of φ I determined by fitting analytical expressions Eqs. (1-5) to the experimental results. All measures applied together allow to identify unambiguously the symmetry class of system and degree of chaoticity. I demonstrated that long-range interaction functions are very sensitive for only 3.5% of missing levels and simultaneously with the high accuracy determine their fraction 1- $\varphi = 0.035$.

To investigate additionally the sensitivity of $S(\tilde{k})$ I generated ensembles of intermediate statistics parameterized by φ for the range $0.7 \leq \varphi \leq 0.965$. Obtained results present a good agreement with analytical curves (Fig.3(b)). Distributions of power spectrum confirm that incomplete spectra provide an interpolation between the Wigner-Dyson (ergodic system) and Poissonian when the fraction of randomly missing levels increases [22-23]. It is known, that the Poisson distribution exhibits uncorrelated energy levels which reveal no repulsion since they fluctuate independently, are randomly distributed and can cross. With an increase of lost levels, the interaction between levels decreases (the Dyson index $\beta \rightarrow 0$). It is worth emphasizing that despite high incompleteness of spectra (about 30%), the measure $S(\tilde{k})$ still delivers complete information about system.

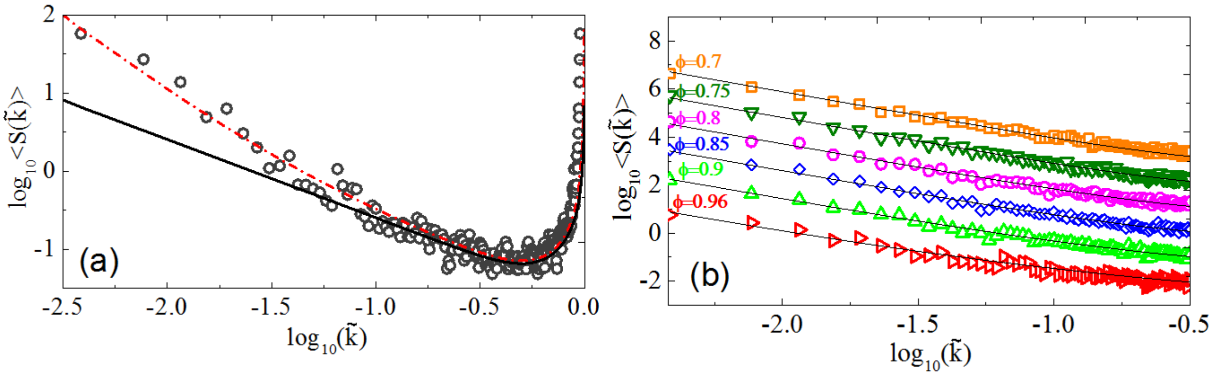


Fig.3** **(a)** Experimental average power spectrum for 30 realizations (black circles). It is compared with predictions for random matrices from the GUE (black solid line) and corresponding missing levels distribution for* $\varphi = 0.965 \pm 0.005$ *(red dash-dotted line). **(b)** Illustration of the power spectrum sensitivity for randomly extracting eigenvalues (curves shifted by unit). Results in **[H1].

We demonstrated that energy-level statistics of quantum system are affected by the breaking symmetry. Moreover, any changes in eigenstates distribution lead to transition between an integrable (φ=0) and chaotic domain (φ=1). The average power spectrum $S(\tilde{k})$ in combination with other measures provide a powerful tool for transparent symmetry characterization and determine the fraction of missing levels 1- φ in complex systems. Problem of incomplete spectra is unavoidable in ensembles of the real physical system that simulates quantum one due to Ohmic losses in microwave coaxial cables and external leads, i.e. the connection to microwave analyzer VNA. The influence of randomly missing levels on correlated spectrum for unitary symmetry was examined in the frequency range 7-14 GHz in the manuscript **[P17]**

This achievement has been announced:

- https://prenumeruj.forumakademickie.pl/fa/2016/12/czy-mozna-odkryc-prawde-o-swiecie/
- https://informacje.pan.pl/14-nauki-scisle-i-nauki-o-ziemi/1158-do-czego-sie-przydaly-sieci-mikrofalowe-czyli-sukces-instytutu-fizyki-pan
- https://naukawpolsce.pl/aktualnosci/news%2C411396%2Ckwantowy-chaos-w-sieciach-mikrofalowych-zbadali-fizycy-z-pan.html

The applicability of correlation functions was also studied for the system with preserved $\mathcal{T}$- invariance **[H2].** The observed deviation from the RMT predictions for long-range spectral fluctuations in this case can be attributed to nonuniversal contributions of short periodic orbits [21-22, 47]. They are localized in the individual bonds of graph as a result of backscattering at the vertices. This phenomenon was investigated for quantum graph and microwave network of the total optical length $\mathcal{L}$=7.04 m that consists of six vertices with the valency of each vertex equaled five and incommensurable lengths of bonds (Fig.4(a)). Analysis were restricted to primitive periodic orbits with the shortest bond of twice length. By varying lengths in steps of 4.2 mm of four bonds with phase shifters, 30 ensembles were measured. Subsequently, I identified approximately 210 resonances for each realization in the frequency range 1-6 GHz obtaining 6300 unfolded eigenfrequencies.

Mainly the long-range spectra fluctuations are affected by periodic orbits, thus the number variance $\Sigma^2(\mathrm{L})$ and the average power spectrum $\mathrm{S}(\tilde{\mathrm{k}})$ were tested. Experimental results (black points) are compared with GOE (solid black line) (Figs.4 (b-c). It is interesting, that the discrepancies with the RMT are clearly perceptible also for complete spectra calculated numerically for closed graph (red lines). In order to study the influence of the shortest periodic orbits on statistical properties of experimental and numerical data, the length spectrum $|\tilde{\rho}(l)| = \left|\int_0^{k_{max}} dke^{ikl}\rho^{fluc}(k)\right|$ was computed, that is the Fourier transform of fluctuating part of the spectral density $\rho^{fluc}(k)$ [48].

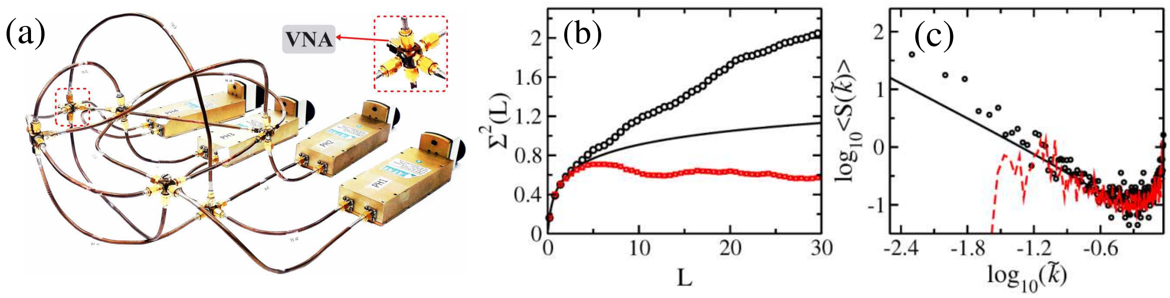


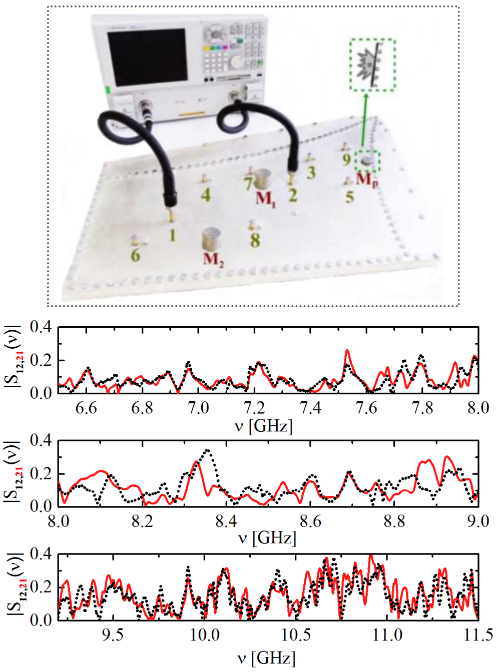
Fig. 4 ***(a)*** *A six vertex microwave network represents GOE systems. Network was coupled to the microwave analyzer VNA (Agilent E8364B) for the scattering matrix measurement. Experimental (black points) and numerical results (red lines) for* ***(b)*** *the number variance $\Sigma^2(L)$,* ***(c)*** *the average power spectrum $S(\tilde{k})$. Photograph and figures from* ***[H2].***

Based on experimental and numerical studies it was shown that appearance of the short periodic orbits in bonds cause a strong deviation from the GOE prediction even for complete spectra. We concluded that deviant behavior in experimental spectra (black points) is attributed only to a small percentage of missing levels. By fitting the analytical Eqs. (2) and (5) to experimental data, I estimated value of φ= 0.965. The impact of periodic orbits doesn't appear in microwave network with violated $\mathcal{T}$-invariance (GUE) since circulators impede the microwave backscattering. Thus, that graph belonging to the Gaussian orthogonal ensemble, even if it contains incommensurable lengths, is not the proper system for the studies of long-range fluctuations due to the presence of shortest periodic orbits. Graphs with GUE, GSE symmetry or two-dimensional microwave resonators are more suitable.

Interpolation GOE → GUE in the RMT model for N-scattering channels [H3-H4]

My great attention has been also devoted to the studies of quantum system with chaotic classical dynamics and partially violated $\mathcal{T}$-invariance **[H3-H4]**. Within the extension of my project "Miniatura 1" (2017/01/X/ST2/00734) I investigated how the degree of time irreversibility can be controlled by magnitude of applied magnetic field. It is characterized by the parameter ξ and determined based on cross correlation coefficients C_{12}^{cross}. Discussed system is modeled by $\mathcal{N}x\mathcal{N}$ dimensional random matrices which are expressed in terms of a sum of a real-symmetric and a real-antisymmetric one $H_{ij} = H_{ij}^{(s)} + i\frac{\pi\xi}{\sqrt{N}}H_{ij}^{(A)}$. Depending on the size of TIV, system interpolates between GOE (ξ= 0) and GUE (ξ= 1) [19].

In experiment I applied the designed and fabricated microwave flat resonator of a quarter bow-tie shape with the area A=1828.5 cm² and the height h=12 mm (Fig.5). The inner surface of aluminum cavity was covered by a 20 μm layer of silver to reduce internal absorption. The top element contains randomly distributed equivalent channels $2 \leq M \leq 9$ represented by microwave antennas of the length 5.8 mm, shunted with 50 Ω loads. System generates chaotic dynamics below the cut- off frequency ν ≈12.49 GHz.



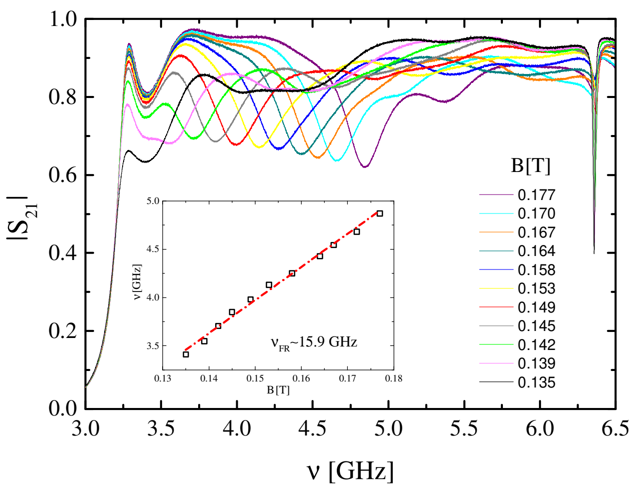


Fig. 5 (left) *Two-dimensional microwave resonator of the "bow-tie" geometry with* $M = 1 \ldots 9$ *channels connected to the analyzer VNA. Cavity contains two ferrites, two pairs of magnets* M_1 *and* M_2 *and perturber* M_p*. Transmission spectra* S_{12} *and* S_{21}*, in three frequency ranges are different* ($S_{12} \neq S_{21}$) *due to partially violated TIV of the magnitude* $\xi \in [0.19\text{–}0.49]$*. Figures presented in* ***[H3].***
Fig. 6 (right) *Ferromagnetic resonances for variable strength of magnetic induction B.* ***Inset:*** *Linear fit determines* ν_{FR}*. Unpublished result.*

In order to violate $\mathcal{T}$–invariance the homogenous magnetic field of induction B≈ 495 mT was generated by two cylindrical NiZn ferrites of the saturation magnetization 2600 Oe (manufactured by SAMWHA, Korea) and magnetized by two pairs of NdFeB magnets. Consequently, the macroscopic magnetization was induced in ferrites that precesses around $\vec{B}$ with the Larmor frequency. It creates the ferromagnetic resonance at $\nu_{FR} \approx 15.9$ GHz, that was determined based on fit (Fig.6 right). In the range $\nu \in [6,12]$ GHz I measured two-port scattering matrix for 100 microwave resonators of varying shape in the function of perturber M_p positions.

The total absorption of electromagnetic wave in walls of resonator is characterized by the parameter $\gamma^{tot} = \frac{2\pi\Gamma(\nu)}{\Delta(\nu)} = \gamma + \eta$, where $\Gamma(\nu)$ denotes the width of resonance and $\Delta(\nu)$ spacing between them [14]. Formula comprises the internal absorption γ and openness of system $\eta = MT_i$, where $T_i=1-|<S_{ii}>|^2$ denotes the transmission coefficient for M scattering channels (for i=1, 2), which is quantified by reflection spectra S_{ii}. Estimated internal absorption $6 < \gamma < 12$ increases with frequency and was controlled by gradually opening channels. Simultaneously I controlled the strength of coupling to the environment, e.g. between the scattering channels and interior region. Coupling is quantified by transmission coefficients T_i (Fig.7(a)). The weak coupling $T_i \approx 0$ corresponds to direct reflection back to the exterior without entering the scattering zone, whereas $T_i=1$ implies perfect coupling [25]. Since ferrite properties strongly depend on microwave frequency, thus degree of $\mathcal{T}$-invariance violation with $\xi \in [0.19–0.49]$ differs with frequency. The strongest TIV $\xi \approx 0.49$ I obtained for M=2 channels in the range $\nu \in [8,9]$ GHz (Fig.7(b)). Nonetheless, above this interval the value is still large $\xi \approx 0.35$. So far partially violated symmetry was obtained for $\xi=0.2$ [49].

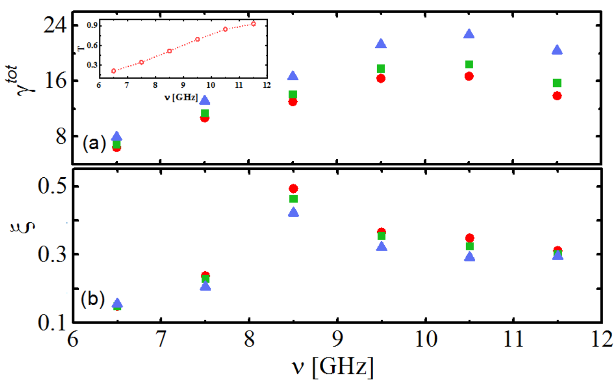


Fig. 7*(a)* *Absorption γ^{tot} for channels: M=2 (red), M=4 (green), M=9 (*blue*),* ***inset****: transmission coefficient T.* ***(b)*** *Respectively the strength ξ of violated symmetry. Results in* ***[H3].***

Further for discussed system I investigated features of scattering matrices which can be quantitatively expressed by the elastic enhancement factor $F_M^{(\beta)}(\eta, \gamma, \xi)$. It measures the enhancement of elastic and inelastic scattering processes with many channels. e.g. scattering of neutrons by protons or nucleus. That is why it plays an important role mainly in nuclear physics [50]. The EEF depends on the universality class denoted by index β, openness of quantum (wave) system η, absorption γ and the magnitude ξ of TIV. It is defined by the ratio of variance of scattering matrix elements [19]

$$F_M^{(\beta)}(\eta,\gamma,\xi) = \sqrt{\langle|S_{aa}^{fl}|^2\rangle\langle|S_{bb}^{fl}|^2\rangle} / \langle|S_{ab}^{fl}|^2\rangle, \tag{6}$$

where $S_{aa}^{fl}(\nu) = S_{aa}(\nu) - \langle S_{aa}(\nu)\rangle$, and $|S_{ab}^{fl}|^2 = \mathrm{C}_{ab}(0;\eta,\gamma,\xi)$ is expressed in terms of $\hat{\mathrm{S}}(\nu)$-matrix autocorrelation function. Limiting values $F_M^{(\beta)}(\eta,\gamma,\xi) = 2$ and $F_M^{(\beta)}(\eta,\ \gamma,\xi) = 1$ are respectively for small absorption γ (well isolated resonances) and large one (strong overlapping). Experimental results (full circles) for M=2, 4, 9 channels lead to the observation that elastic enhancement factor decreases with $\mathcal{T}$-invariance violation increasing (Fig.8(a)). Whereas, it increases with the openness increasing (when M=2 → 9). While ν > 6 GHz, the EEF is below 2 and displays a minimum in the interval ν ∈ [8,9] GHz.

Based on such behavior we formulated conclusion that effect of $\mathcal{T}$-invariance violation is dominative over the openness. It is worth remarking, that behavior of channels with violated TIV is peculiar since they are oppositely distributed in comparison with the case for preserved $\mathcal{T}$-invariance (empty circles for ξ=0). Results for $\vec{B}$=0 are shown in **[P15].**

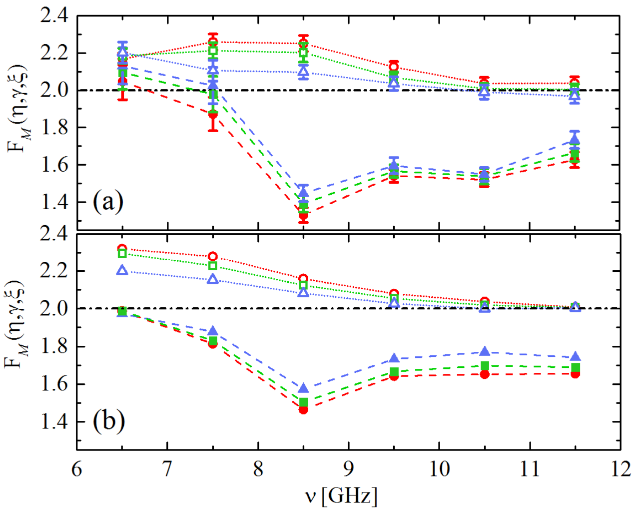


Fig. 8(a) *Experimental elastic enhancement factor with the standard deviation denoted by error bars, averaged in 1 GHz window over 100 realizations of cavities for M=2 (red), 4 (green), 9 (blue) scattering channels. Full symbols correspond to the presence of magnetic field $\vec{B}$ inside cavity, whereas empty points to preserved $\mathcal{T}$-invariance (ξ=0).* ***(b)*** *Respectively EEF for RMT. Results presented in **[H3]**, results with $\vec{B}=0$ in **[P15]**.*

The behavior of experimental enhancement factor was confirmed by theoretical studies based on random matrix theory (Fig.8(b)). Numerical calculations are in good agreement with experimental ones as they also confirm reversed sequences of channels. Thus, it was exposed that size of symmetry violation can effectively modifies openness η of system. Since the system is strongly sensitive to openness, the proper methodology of measurement is desirable.

Interesting properties of quantum system with interpolation GOE → GUE I investigated in more detail considering also spectral correlation functions of scattering matrix **[H4]**. Analysis of interaction functions in such system is a challenging task, since besides the intermediate dynamics between Gaussian orthogonal and unitary ensembles, additionally the incomplete sequences of levels must be taken into account. Then short- and long-range level interactions depend also on the parameter ξ. NNS distribution for λ=2ξ is well approximated by the formula [51]

$$P(s,\lambda) = s\sqrt{\frac{2+\lambda^2}{2}}\,c(\lambda)^2 \mathrm{erf}\,(s\frac{c(\lambda)}{\lambda})e^{-\frac{s^2c(\lambda)^2}{2}} \tag{7}$$

where erf (x) denotes the error function and $c(\lambda) = \sqrt{\pi\frac{2+\lambda^2}{4}}\left[1 - \frac{2}{\pi}\left(tan^{-1}\left(\frac{\lambda}{\sqrt{2}}\right) - \frac{\sqrt{2}\lambda}{2+\lambda^2}\right)\right]$. Whereas, interpolating formulas for long-range measures are expressed by the two-point cluster function [52-53]

$$Y_2(L,\xi) = det\begin{pmatrix} s(L) & -D(L,\xi) \\ -J(L,\xi) & s(L) \end{pmatrix} \tag{8}$$

with $s(L,\xi) = \frac{sin\pi L}{\pi L}$, $D(L,\xi) = \frac{1}{\pi}\int_0^{\pi} dx e^{2\xi^2x^2} xsin(Lx)$ and $J(L,\xi)$=$\frac{1}{\pi}\int_{\pi}^{\infty} dx e^{-2\xi^2x^2}\frac{\sin(Lx)}{x}$.

It is known, that the cumulative number of resonances is expressed as $N(\nu){\sim}\frac{A\pi}{c^2}\nu^2$, where A is the area of resonator. Thus the density of resonances increases with frequency results in overlapping that disenables identification of all resonances (is time consuming task).

Figure 9 illustrates the applicability of analytical expressions (7-8) for statistical measures of short- and long-range interactions. All results for N=2 channels are averaged over 25 realizations and presented in the three frequency ranges, for φ and ξ. The nearest-neighbor spacing distribution P(s) exhibits slight deviation of experimental data from theoretical curves. Whereas, for long-range interaction functions I obtained a very good agreement. The deviations from corresponding theoretical curves predicted in the RMT are observed (red dashed lines) due to incompleteness of spectra and strength of TIV. I verified parameters φ and ξ using the spectral rigidity Δ_3(L), number variance Σ^2(L) and average power spectrum $\langle S(\tilde{\tau})\rangle$, that for $\tilde{\tau} \gg 1$ is very sensitive on the fraction of missing levels. Thus it provides a powerful measure to determinate them. All results are consistent with the elastic enhancement factor $F_M^{(\beta)}(\eta, \gamma, \xi)$ presented in **[H3]**.

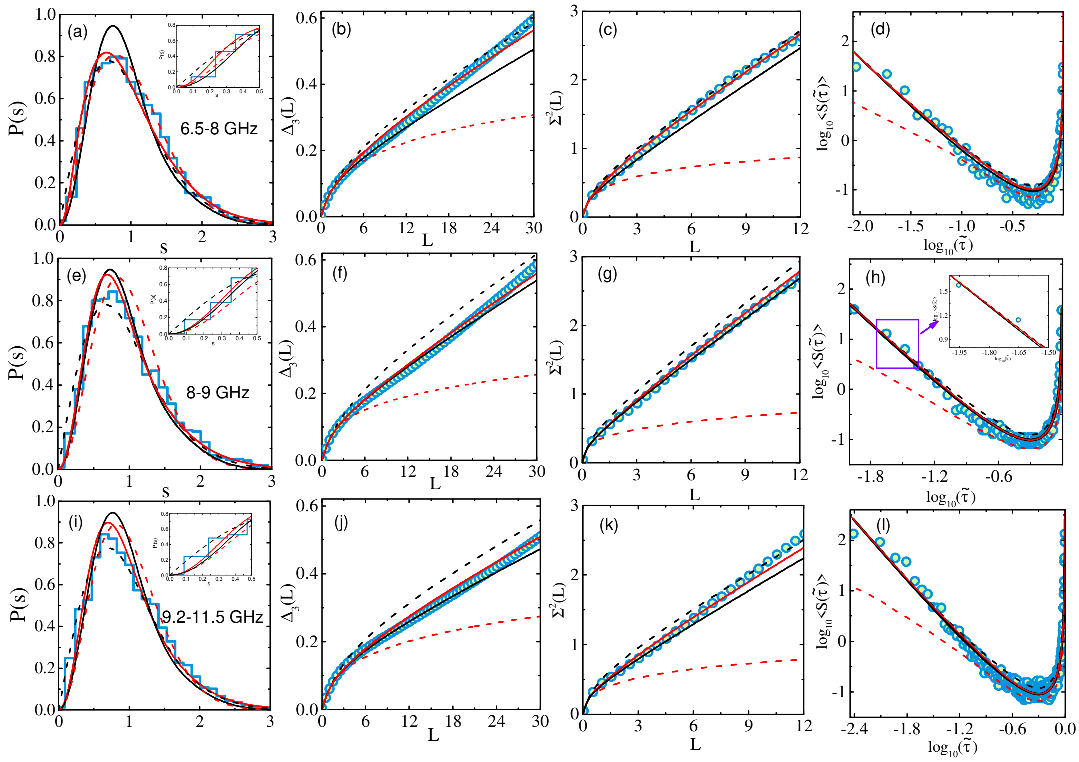


Fig. 9 *Results for the three frequency ranges, for different φ and ξ, (in rows) and respectively: the nearest neighbor spacing distribution P(s), the stiffness Δ_3(L), the number variance Σ^2(L) and the average power spectrum $\langle S(\tilde{\tau})\rangle$. Experimental results (blue histograms and points) are compared with the RMT curves for intermediate case between GOE and GUE (red dashed line) considering φ (red solid line). The black dashed and solid lines correspond respectively to GOE and GUE for the proper φ. Results in **[H4].***

ν [GHz]	ξ	φ	N	N_{total}
6.5 – 8	0.19	0.83 ± 0.03	110	2750
8 – 9	0.49	0.81± 0.03	90	2250
9.2 –11.5	0.35	0.85 ± 0.03	258	6450

Tab. *Parameters for the three frequency ranges.*

I showed that statistical analysis of spectral properties deliver precise information about quantum systems, provided that the combination of different functions is applied. Then they allow to assign unambiguously the strength of broken symmetry ξ and the fraction 1-φ of undetected eigenfrequencies. Systems with interpolation GOE → GUE reveal an increase of level repulsion and becomes more correlated. Nonetheless, on the other hand when φ→ 0 (the incompleteness of spectra increases), the degree of correlation decreases which provide an transition in opposite direction, i.e. from Wigner-Dyson (φ=1) to Poisson (φ=0) statistics. Due to complex behavior of quantum (wave) system it is desired to use carefully statistical measures and interpret them. I illustrated how to use them properly to obtain unambiguous information about system, since their analysis might be sometimes disputable [54].

Pseudo-integrable system with the semi-Poisson statistics [H5]

The experimental realization of integrable system with uncorrelated spectra (β=0) is a difficult in microwave billiard, since wire antennas which are coupled to the power supplier VNA change the spectral properties of scattering system. Nonetheless, in **[P17]** we demonstrated that for rectangular cavity with a week perturbation, statistics in the range $v \in [3.8 - 8]$ GHz are very close to the Poisson. For higher frequency the deviation towards a semi-Poisson distribution is observed that exhibits interaction only between adjacent levels. Specific properties of intermediate system between Poisson and Wigner-Dyson, that is pseudo-integrable (the so called critical) we investigated in **[H5]**.

This system was simulated by a rectangular microwave cavity fabricated from brass and perturbed by two point-like scatterers (antennas). The adjustable length L_1 = 36.5-41.5 cm was changed in 25 steps of measurement for each pair of antennas, while the second width L_2=20.2 cm was constant. The height of d=8 mm corresponds to cut-off frequency $\nu_{\max}= c/2d$ ≈18.7 GHz (Fig.10(a)). Two microwave antennas of the length 3 mm and diameter 0.9 mm were introduced into the cavity and coupled to the vector network analyzer Agilent E8364B through two microwave cables. They perform the role of two M =2 scattering channels in the measurement of the two-port scattering matrix $\hat{S}(\nu)$. For spectral statistics in the frequency interval 8-13.5 GHz I identified 9224 resonance frequencies for 30 cavity realizations.

To examine the pseudo-integrable system I applied the short range plasma model. Experimental NNS distribution (histogram) is compared with the theoretical curves for Poisson (green dotted line), semi-Poisson (red full line) and GOE (blue dash- dot line) (Fig.10(b)). Inset demonstrates the integrated P(s). Based on fit (black circles) given by formula [15]

$$P(s,\eta) = \frac{\eta^{\eta} s^{\eta-1} e^{-\eta s}}{\Gamma(\eta)} \tag{9}$$

to the experimental data I determined parameter η=1.972 ± 0.049. It characterizes the departure from regularity, where $s \geq 0,\ \eta \in [1, +\infty]$, and $\Gamma(z) = \int_0^{\infty} dt\ t^{z-1} e^{-t}$ is Gamma function. For the Poisson distribution η=1 and semi-Poisson η=2. Fig.10(c) exhibits experimental nearest-neighbor spacing distribution compared with the theoretical one of the second order $P^{sP}(2,s) = \frac{8}{3}s^3e^{-2s}$ (violet full line) [55], that confirms the semi-Poisson behavior.

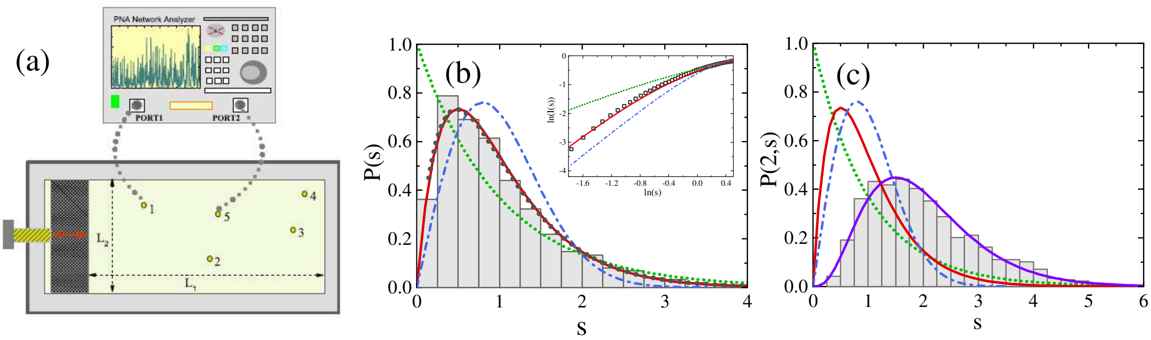


Fig. 10 ***(a)*** *Schematic view of two- dimensional rectangular microwave resonator that simulates a quantum billiard with the semi-Poisson distribution. PORT1 and PORT2 are attached to two microwave antennas, the side L_1 is movable.* ***(b)*** *Experimental NNSD (histogram) is compared with the semi-Poisson (red full line). Poisson and GOE distributions correspond respectively to green dotted line and blue dash-dot line. Fit of the formula (9) is marked by black circles.* ***(inset)*** *Integrated NNSD.* ***(c)*** *Experimental second order of NNSD (histogram) compared with the theoretical prediction (violet line). Figures in* ***[H5].***

Since the elastic enhancement factor $F^{sP}(\gamma^{tot})$ was the priority in the studies of semi-integrable system, the analytical formula was derived [56]

$$F^{sP}(\gamma^{tot}) = 3 - \frac{\gamma^{tot}}{\pi}\left[\mathrm{ci}\left(\frac{2\gamma^{tot}}{\pi}\right)\sin\left(\frac{2\gamma^{tot}}{\pi}\right) - \mathrm{si}\left(\frac{2\gamma^{tot}}{\pi}\right)\cos\left(\frac{2\gamma^{tot}}{\pi}\right)\right] \tag{10}$$

where $\mathrm{si}(x) = -\int_0^\infty \frac{\sin(t)}{t}dt$ and $ci(x) = -\int_0^\infty \frac{\cos(t)}{t}dt$. Neither, experimental nor theoretical results for EEF in the semi-Poisson regime had been presented so far, thus they are shown for the first time in **[H5].** Figure 11(a) demonstrates dependence of enhancement with standard deviation (black points with error bars) on γ^{tot} and ν, in the range $\nu \in [8 - 13.5\,]$ GHz. It was calculated within 25 MHz sliding window and averaged over 150 realizations. Result is compared with theoretical curve (red full line) expressed by the analytical equation (10). Two broken lines are attributed to the asymptotes of the semi-Poisson statistics. Therefore, enhancement factor diminishes gradually from $F^{sP}(\gamma^{tot}) = 3$ to $F^{sP}(\gamma^{tot}) = 2.5$, respectively for very small and large γ^{tot}. For comparison Fig. 11(b) depicts EEF for the GOE in RMT approach (blue line).

Enhancement factor $F^{sP}(\gamma^{tot})$ corroborates the pseudo-integrable behavior of system. Simultaneously the semi-Poisson model provides an characterization of level statistics for the short- and long- range correlations. In **[P21]** we studied EEF in a transient region between regular and chaotic dynamics.

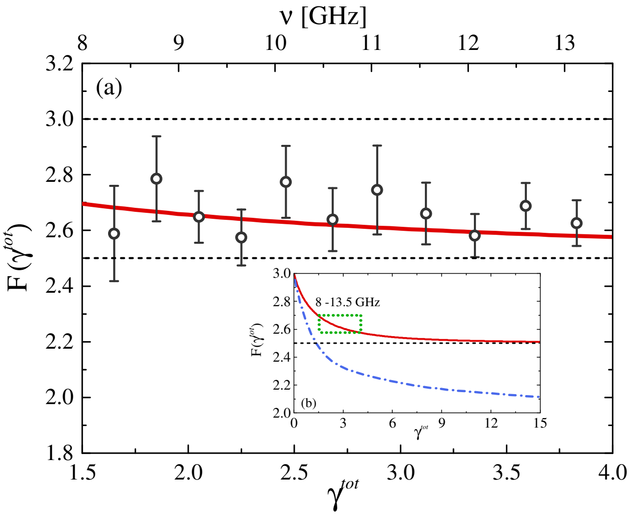


***Fig. 11(a)** The experimental elastic enhancement factor with standard deviation (black points with error bars) compared with the theoretical one (red solid line). Two broken lines indicate limits for the semi-Poisson behavior. **(b)** Blue line corresponds to GOE distribution while green rectangle shows analyzed frequency window 8–13.5 GHz. Results in **[H5]**.*

Applicability of spectral correlation functions in 3D system [H6]

The impact of strong resonances overlapping on applicability of correlation functions I studied in a three dimensional resonator (studied occasionally), i.e. when the average resonance spacing is small compared with their width Δ<<Γ **[H6]**. Only a very few experimental studies have been devoted to these systems due to experimental difficulty. The number of levels in 3D system according to the Balian-Bloch formula exhibits the cubic dependence on the frequency $N(\nu)\sim\nu^3$ [57]. Because of the strong increase of eigenstates density, many eigenfrequencies can not be identified, causing the high fraction of missing levels 1-φ. Then short- and long-range level correlations became useless. Thus, the symmetry class and chaoticity of system can be verified based on the elastic enhancement factor $F_2^{(1)}(\gamma)$, as it does not depend on the average spacing between levels.

Three dimensional system was simulated experimentally in a half- cylinder microwave cavity of the volume $V=7.267\mathrm{x}10^{-4}$ m (Fig.12 (a)). The rough wall of mean radius R_o = 10 cm adjacent to the convex and inclined bottom plate is described by the radius function $R(\theta) = R_o + \sum_{m=2}^{\mu} a_m \sin(m\theta + \Phi_m)$, where μ=20, $a_m \in [0.084,0.091]$, $\Phi_m \in [0,2\pi]$ and $0 \leq \theta < \pi$. Two ports out of three randomly distributed, with antennas of the length 6 mm were attached to VNA while the third port was closed with a plug. In the measurement of two-port scattering matrix $\hat{S}(\nu)$ with the scatterer rotated in steps of 10^o, for each combination of antennas position, 107 configurations of resonator were recorded.

For 3D system I analyzed more complicated scenario, when levels are missing not only randomly, but also systematically. Figures 12 (b-c) illustrate that delta statistics $\Delta_3(L)$ and power spectrum $S(\tilde{k})$ deviate from the theoretical predictions for the GOE in RMT. Experimental results for 2700 eigenfrequencies (black points) were averaged over 30 realizations in the range $\nu \in [13-14]$ GHz. The curves for the GOE in RMT predictions and for 74% of identified levels are marked respectively by solid black and red dashed–dotted lines. Based on numerical simulations of random matrices from the GOE (green points) I estimated 11% of randomly missing levels. Further 15% I attributed to systematically missing neighboring eigenvalues. For $\varphi = 0.74$, while strong overlapping with the high fraction of systematically missing levels takes place, short- and long- range correlation functions can't be applied for symmetry and chaoticity identification. However, as I depicted in **[H1],** long-range correlation functions may be applied even for 30% missing levels, provided they are lost randomly.

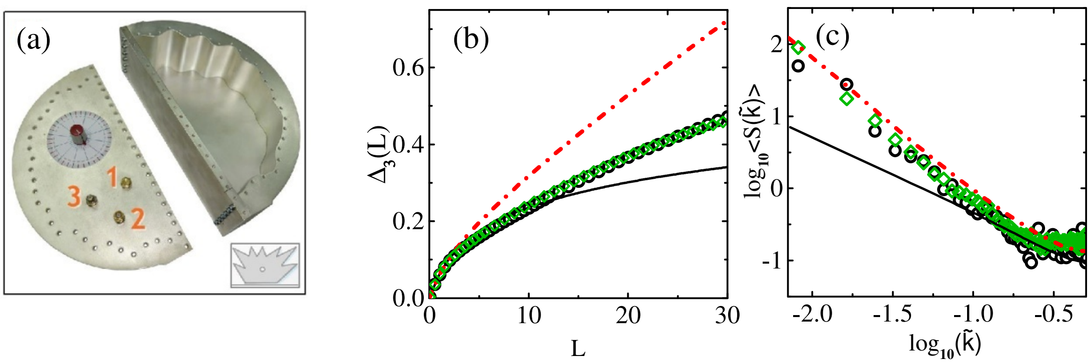


Fig.12 ***(a)*** *3D microwave resonator with marked position of antennas and scatterer. Experimental results (black points) for the long-range correlation functions* ***(b)*** *delta statistics* $\Delta_3(L)$ and ***(c)*** *the power spectrum* $S(\tilde{k})$. *Results are compared with numerical simulations (green points). The RMT curves for GOE and the fraction* φ*=0.74 are marked respectively by black solid and red dash- dotted lines. Results in* ***[H6].***

Fig. 13 (a) illustrates the experimental elastic enhancement factor for 107 shape of cavities and averaged in 1 GHz sliding window. Average value of $F_2^{(1)}(\gamma) \cong 2.08 \pm 0.10$ (red line with band) was calculated based on the expression (6) and compared to numerical ones (blue points). Results are consistent and corroborate that open 3D cavity exhibits properties of fully chaotic system with preserved $\mathcal{T}$-invariance. EEF doesn't require complete sequences of spectra, therefore it may be applied to specify the chaoticity and symmetry of system even for the high frequency $\nu \in [11-25]$ GHz, in regime of strongly overlapping resonances. Whereas, this information cannot be verified based on level correlation if they are missing systematically. A good coupling of system characterized by transmission coefficients of two scattering channels T_1 and T_2, and the large total absorption $5 < \gamma < 15$ averaged in 1 GHz window are shown in Figs.13 (b-c).

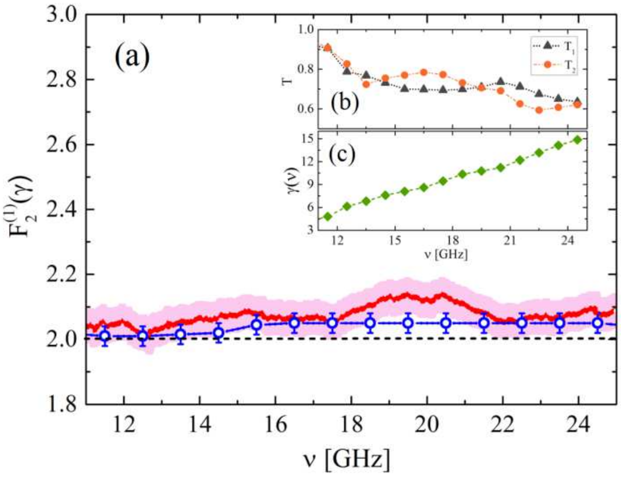


Fig.13(a) *The experimental elastic enhancement factor with standard deviation averaged over 107 realizations in 1 GHz sliding window (respectively red solid line with band) compared with the numerical one (blue points averaged in 1 GHz window).The black dashed line corresponds to the RMT limit of the EEF for strong absorption.* ***inset:*** ***(b)*** *The transmission coefficients for 2 channels* **(c)** *Total absorption strength. Results in* ***[H6].***

Spectra sensitivity to local transformations of system [H7]

Publication **[H7]** is devoted to the studies of spectra sensitivity to local transformations of graph, the so-called "switch". This operation is realized by exchanging one pair of edges linked to a common vertex [19]. Although edge lengths and boundary conditions are unaltered, the impact of such operation on spectra is observed. Spectra after modification $\{\widetilde{E_n}\}_{n=1}^{\infty}$ is degree-r interlaced with the original one $\{E_n\}_{n=1}^{\infty}$, if $E_{n-r} \leq \widetilde{E_n} \leq E_{n+r}$, where $n > r$. Consequently, the eigenvalue of "switched" spectra appears between two neighboring eigenvalues of the original one.

This transformation I investigated for fully connected 4-vertices tetrahedral microwave network. To realize system with the orthogonal symmetry class (β=1) we used network of the total optical length $\mathcal{L}$=2.248 ± 0.008 m connected to one port of VNA analyzer (Fig.14(c) left insert). The measurement of reflection in the frequency interval 0.01-2.5 GHz was conducted by decreasing the lengths of phase shifter PHA, while PHB was increased with the same step 10°. Figures 14 (a-c) depict modulus of the scattering matrix for original $|S(\nu)|$ and switched $|\tilde{S}(\nu)|$ network, and counting functions $N(\nu)$ before and $\tilde{N}(\nu)$ after transformation. Network of the length $\mathcal{L}$=2.918 ± 0.010 m with violated $\mathcal{T}$-invariance (β=2) (Fig.14 (c) right inset) was measured in the range 0.8-2.5 GHz due to circulator characteristic. Number of resonances in both cases was in agreement with the Weyl's formula $N(\nu) = 2\mathcal{L}\nu/c$ [40].

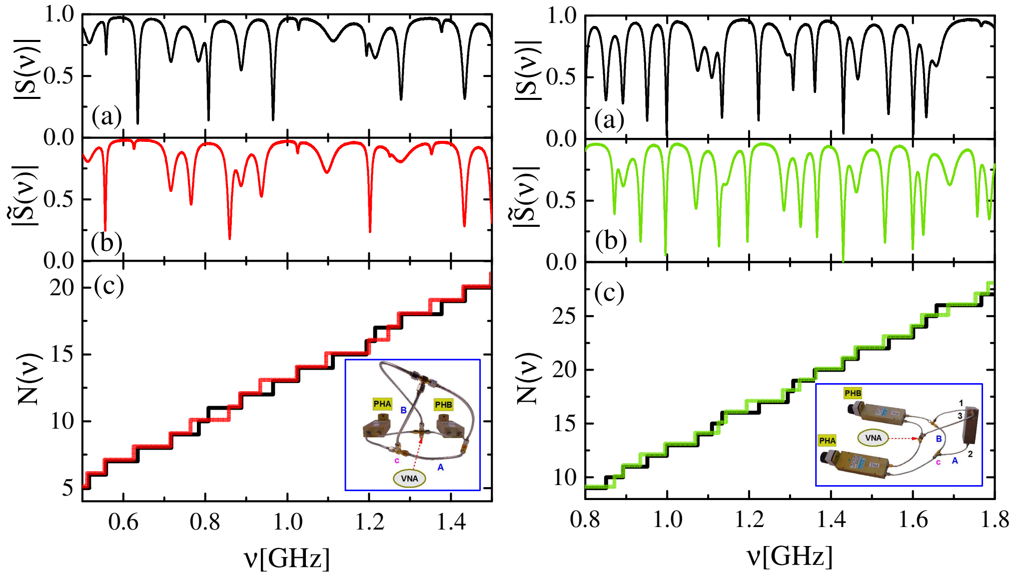


Fig. 14 Left GOE (a)-(b) *The modulus of scattering matrix measured for the microwave network before the edge "switch" transformation* $|S(\nu)|$ *and after this operation* $|\tilde{S}(\nu)|$. ***(c)*** *Counting functions N(ν) for original spectra (black line) and switched* $\tilde{N}(\nu)$ *(red line).* ***Inset:*** *Microwave networks simulating quantum graphs with preserved* $\mathcal{T}$*-invariance.* ***Right****: The same for GUE. Results in* ***[H7].***

We analyzed distribution P(ΔN) of the spectral shift ΔN=$N(\nu)-\tilde{N}(\nu)$. Experimental results for P(ΔN) to be ±1 or 0 (black bars) were compared with the numerical ones (red bars) for GOE (Fig.15(a)). They reveal that approximately 12% of levels are sensitive on local modifications of system results in changes of level correlation. Similarly for GUE in Fig.15(b). The instance of changing $\Delta N \geq 2$ characteristic for "swap" transformation is not observed since the spectra under edge "switch" operation are level-1 interlaced (r=1) with that of the original one for both the GOE and GUE. However the topology of graph was preserved, spectra are interlaced. This proves how sensitive they are to the systems modifications. Experimental and numerical results confirm theoretical predictions [19]. This transformation may be useful to identify missing resonances.

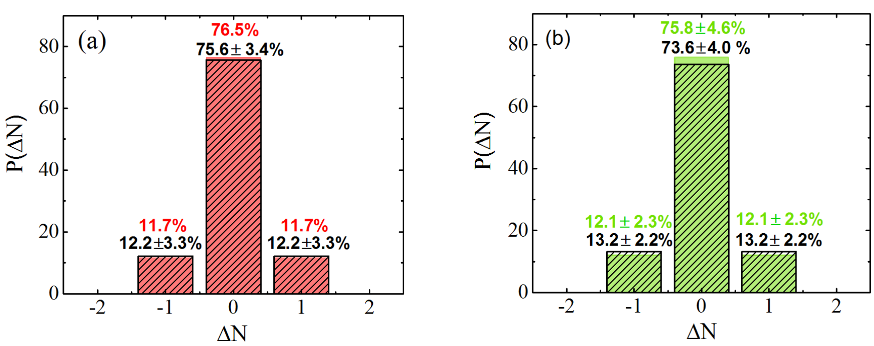


Fig.15 (a) *Distribution P(ΔN) of the spectral shift ΔN for GOE, experimental results (black bars) are compared with the numerical ones (red bars).* ***(b)*** *Respectively for the GUE. Results in* ***[H7].***

2.4 Impact of my achievements on the discipline development

The series of publications **[H1-H7]**, which constitute my scientific achievement, encompass the research of spectral properties in low-dimensional quantum systems of different degree of levels correlation:

- modeling of one- and two-dimensional quantum systems in microwave experiments (also three-dimensional system),
- analysis of one- or two-ports scattering matrix,
- the application of spectral correlation functions considering incompleteness of spectra, impact of internal absorption, the strength of coupling with external environment, influence of N scattering channels, periodic orbits, local transformations of system.

1) Spectral properties of experimentally simulated system with violated time-reversal ($\mathcal{T}$) symmetry for the first time were presented in our publication **[H1**]. The power spectrum $S(\tilde{k})$ was incorporated for which I studied the sensitivity on the fraction of randomly missing levels, while $\varphi \rightarrow 0.7$. It results in systems transition from GUE towards Poisson. I showed that short-range correlation P(s) in combination with long-range correlations in terms of the number variance $\Sigma^2(L)$, the spectral rigidity $\Delta_3(L)$ and average power spectrum $S(\tilde{k})$ provide strong tools in the study of complex dynamics of system. Consistent results allow to distinguish between different degrees of chaoticity in the random matrix theory (RMT) and determine the fraction of missing levels with the high accuracy even if spectra are strongly incomplete**.** Analysis reveal how to extract complete information about quantum (wave) system.

2) I studied the applicability of correlation functions for the simulated system with preserved $\mathcal{T}$- invariance. It was found out experimentally and numerically that the deviation from the RMT predictions for long-range spectral fluctuations can be attributed to the nonuniversal properties of graph with GOE symmetry class, that is the contribution of short periodic orbits arising by reflection in the vertices **[H2]**.

3) A complex system in transition region GOE → GUE as a result of partially violated time-reversal ($\mathcal{T}$) symmetry I studied in **[H3]** considering additionally the impact of controllable openness of resonator with N-scattering channels. Obtained results reveal how the strength of violated symmetry (TIV) can modify openness and how this affects the elastic enhancement factor $F_M^{(\beta)}(\eta, \gamma, \xi)$. The distribution of enhancement factor for N-channels deserves attention as it is reversed in contrast to distribution for $\vec{B} = 0$.

4) Properties of system with partially broken symmetry I studied using also spectral correlation functions for N=2 channels and simultaneously taking into account the fraction of randomly lost states 1-φ **[H4]**. Due to observed interpolation GOE → GUE, the degree of level correlation increases. On the other hand, because of missing levels (φ→0), the interaction between them vanish and system exhibits deviation in the opposite direction, i.e. from the Wigner-Dyson to integrable regime. I observed the greatest departure from RMT for long-range interactions, same as for the GOE and GUE. Applied measures revealed how to study complex dynamics of system when correlation functions depend on many parameters. The precise analysis of eigenvalues in spectra is significant and the proper unfolding procedure.

5) A pseudo-integrable system of unique properties with the semi-Poisson statistics I investigated in **[H5]**, demonstrating how it can be realized experimentally. It corresponds to critical region between Poisson and GOE. For the first time the elastic enhancement factor $F^{sP}(\gamma^{tot})$ for the semi-Poisson was presented. Additionally I verified this measure applying the short-range plasma model and determined parameter η that characterizes deviation from an integrability.

6) In three-dimensional ergodic system with the Gaussian orthogonal ensemble (GOE) I examined the influence of systematically lost states (not only randomly), caused by the high openness of system **[H6]**. Experimental and numerical results corroborated that spectral correlation functions are no longer applicable. Then the elastic enhancement factor allows to determine symmetry class in RMT, even for very strong resonances overlapping.

7) For quantum graphs with the Dyson index β=1 and β=2 simulated by tetrahedral microwave networks, I demonstrated that spectra under local "switch" modification are interlaced of degree-1 in a frequency domain, nonetheless topology of graph is preserved **[H7]**. This transformation reveals how spectra are sensitive and consequently affects level correlation. Experimental and numerical results confirmed theoretical predictions.

8) The results presented in publications [**H1-H7]** are innovative and important for detailed analysis of spectroscopic data. They are desired and precious since the investigation of quantum systems have been performed mostly theoretically, whereas the high-resolution experiments are conducted barely by few teams around the world. One of them is the group of prof. dr hab. Leszek Sirko, the leader of quantum and wave chaos phenomena investigation. All results were prepared for a large number of ensembles, which additionally increase their value from the statistical point of view. Due to interdisciplinary character of our research, results are also important for other areas, such as chemistry, biology or quantum information.

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3. Description of another scientific achievement [D1-D3]

At the Faculty of Physics, Warsaw University of Technology (2001-2011) I carried out scientific research composed of two stages:

I stage: participation in the construction of Laboratory of Femtosecond Techniques at the Faculty of Physics, WUT

1) I participated in the construction from scratch of modern laboratory with an original Electro-Optic Sampling System (EOS), that was a great challenge. This is an advanced tool for characterization of ultra-high-speed detectors and optoelectronic elements with sub-picosecond resolution. The necessity to build EOS setup was caused by the fact that the main objective of my research referred to the investigation of the fastest existing photodetectors with time response signal in THz domain, which were fabricated in technology elaborated by the Research Center Jülich (Germany). The former laser of laboratory construction was not adapted to research with the high precision.

2) The femtosecond Ti:sapphire laser of the newest generation, Spectra-Physics Tsunami HP, was purchased (Fig.16(a)). This is utilized in the studies of ultrafast processes and was a desirable element in the EOS setup. The design of other components and construction of system (Fig.16(b-d)) was mainly my duty since it was a pioneering experiment in Poland using this system and my research was the first realization.

3) The idea of the EOS system operation encompass the sub-picosecond technique of pump-probe spectroscopy with a controlled delay line and linear Pockels effect, making the measurement method difficult. Conceptually this setup was similar to the one operating at the University of Rochester (USA), nonetheless I incorporated few changes. The most important was the modification of pyramidal geometry of electro-optic $LiTaO_3$ crystal to improve the sampling of electric pulses. I also elaborated the methodology of ultra-short response detection. Further I verified metrological properties of setup and optimized impact of its various parameters on electro-optic signals.

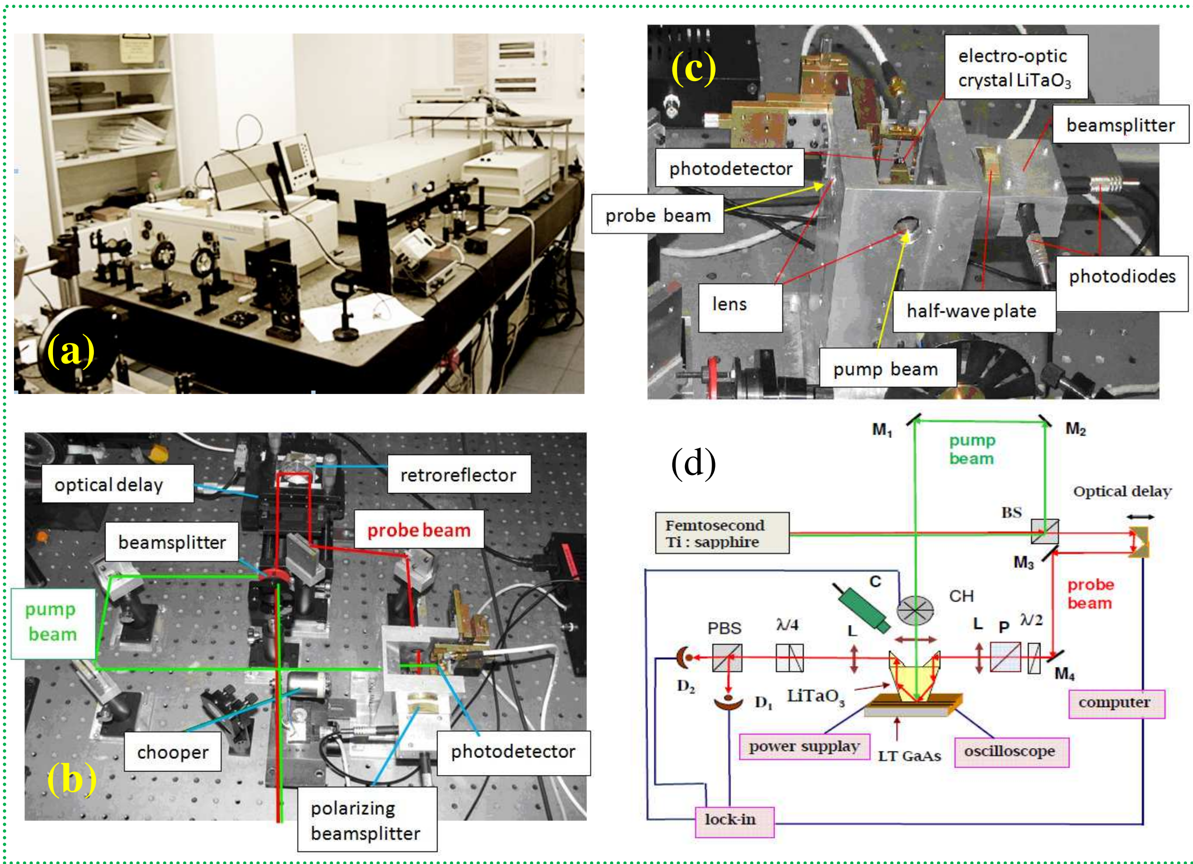

***Fig. 16** Constructed Electro-Optic Sampling System (EOS) at the Faculty of Physics, WUT **(a)** Femtosecond laser Ti: sapphire (Spectra-Physics Tsunami HP). **(b)** Depicted pump-probe technique (with optical delay), green and red lines- the path of pump and probe beams respectively. **(c)** Detection system with an investigated photodetector and electro-optic crystal $LiTaO_3$. My photos. **(d)** EOS scheme in[D2]*

II stage: Investigation of ultrafast LT GaAs photodetectors

1) I formulated a research problem and afterwards examined the sensitivity and time resolution for *LT* GaAs photodetectors. Microstructures of low temperature GaAs, metal-semiconductor-metal (MSM) with carrier lifetime < 200 fs were fabricated in the MBE technology (Molecular Beam Epitaxy) at the Research Center Jülich (Fig.17(a)). In electro-optic measurement I obtained a pulse with FWHM of ~ 980 fs (f_{-3dB} ~0.55 THz). Comparable time was measured at the University of Rochester (USA) where detector was also tested. This allowed to verify that constructed system and applied by me methodology and my conception of measurement works properly. In experiments the femtosecond laser beam of 80 fs pulse duration, 795 nm wavelength and 80 MHz repetition was used.

2) Another type of N^{+}GaAs photodetector was additionally irradiated by fast neutrons at the National Centre for Nuclear Research in Świerk results in response reduction up to~680 fs.

3) From the Research Centre Jülich I received on my request photodetectors of the newest generation, i.e. *LT* GaAs integrated with coplanar transmission lines (CPS) (Fig.17(b)). Its ultra-short response time with FWHM of ~720 fs (f_{-3dB} ~0.74 THz) that I measured was attributed to the fastest photodetectors manufactured so far (Fig.17(c)). The available at that time commercial detector (Hamamatsu) had the resolution barely ~ 40 ps.

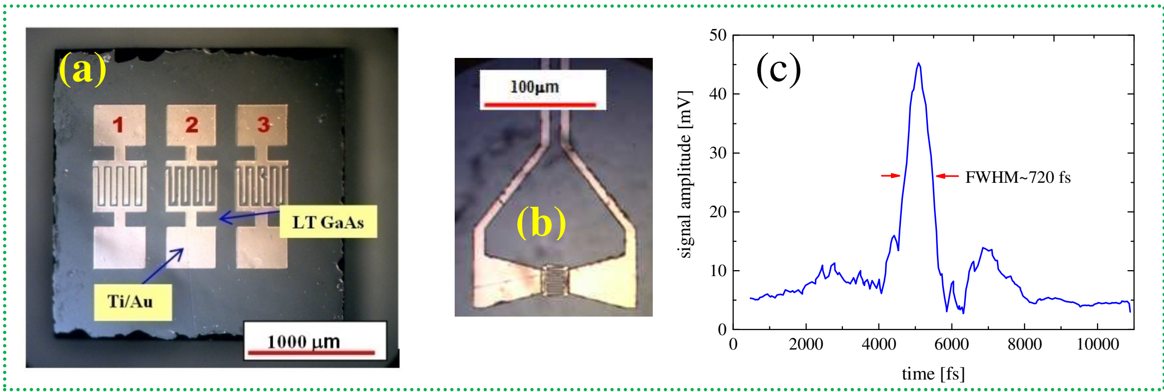

***Fig. 17 (a)** LT GaAs photodetectors of the three different geometry with Ti/Au metallization **(b)** Photodetector LT GaAs integrated with CPS. **(c)** Response time of structure with CPS. Detectors received from the Research Centre Jülich, Germany. My photos and result.*

Impact of my achievements on discipline development:

1) Constructed EOS system contributed to the fact that newly created Laboratory of Femtosecond Techniques was one of the most modern at the Faculty of Physics, WUT.

2) Built setup initiated in Poland the research of ultrafast photodetectors, allowed to conduct research in the area of emission and detection of terahertz radiation and realize further doctoral and master theses.

3) System has become an alternative in Europe to the system operating in the USA, where prof. R. Sobolewski's group from many years is the world leader in this area.

4) I opened the possibility for cooperation with the Research Center in Jülich, Germany, also with VIGO System S.A., one of the best innovative companies in the optoelectronics branch, that delivers own detectors inter alia for the NASA.

5) I demonstrated an effective method of reducing the photodetectors time resolution by applying neutron irradiation, that is an alternative method to the common nitrogen ion implantation.

6) My contribution to discipline development was significant and innovative due to an intensive research of detectors and optoelectronic devices in recent years and rapid development of terahertz techniques.

Importance of research and my activity according to prof. dr hab. Józef Piotrowski, the reviewer of my doctoral thesis and founder of VIGO System S.A. (quotes from the review):

- ***s*cientific issues considered in the work:** *"the conducted research can be related to femtosecond photonics, one of the fastest-growing fields of technical science currently of great cognitive and application importance, the topic of the dissertation is therefore important and current", "in conducted research the theoretical methods were used - analysis, comparison and synthesis, as well as experimental methods - construction of advanced research system, its characterization and then testing properties of ultrafast radiation detectors",*
- **in the context of sources analysis, including world literature, and the state of knowledge**: *"the review of the state of knowledge relating to the subject of research is performed competently, appropriate conclusions were drawn and used in own research", "the quality of the review proves that the author is properly prepared to conduct research in the field of optoelectronics, in particular in the theory, construction and technology of radiation detectors, as well as in advanced methods of their characterization".*
- **did the author solve the problems, did use the right method:** *"The author of the dissertation demonstrated the ability to solve complex problems in the field of advanced optoelectronic technologies",*

- **originality of the dissertation, independence and original work of the author, position of the dissertation in relation to the state of knowledge and the level of technology presented by world literature**: *"system described in the dissertation is the first Polish construction of a system for measuring ultrafast detectors using the electro-optic sampling method. Applied solutions are similar to those used in the system of the well-known center in Rochester (USA) and have characteristics comparable to the best devices of this type developed so far", "construction of the setup needed to overcome significant difficulties",*
- **usefulness of dissertation for technical sciences**: *"results presented in the dissertation are valuable", "the developed research techniques present significant value for the further development of semiconductor optoelectronics. They can be used to characterize femtosecond radiation detectors in various spectral ranges, in particular to characterize infrared radiation detectors manufactured in Poland".*

Publications affiliated with the Faculty of Physics, Warsaw University of Technology:
results and manuscripts prepared with my dominant participation

[D1] M. Białous, R. Mogiliński, M. Wierzbicki, K. Świtkowski, B. Pura
Comparison of ultrafast photodetectors based on N^+GaAs and LT GaAs,
Acta Physica Polonica A no **3**, vol 119 (2011)

[D2] M. Białous, M. Wierzbicki, B. Pura, K. Brudzewski, J. Strzeszewski
Subpicosecond photodetector N^+GaAs irradiated by fast neutrons,
Applied Physics B **96** (2009)

[D3] M. Białous, K. Świtkowski, A. Kozanecka-Szmigiel, B. Pura, M. Wierzbicki
Investigations of photoresponse signals of LT GaAs photodetector,
Optica Aplicata, issue **4**, vol.39 (2009)

4. Significant scientific activity carried out at more than one university or scientific institution

I carried out research in three leading centers, referred successively to three areas: atomic physics, solid state physics (optoelectronics), and aspects of quantum mechanics. I also collaborated with national and foreign centers.

- **Institute of Physics, University of Nicolaus Copernicus in Toruń (1992/1993) -** studies in selected group of only few persons required scientific activity, moreover I participated in an advanced experiment in the laboratory of Atomic Physics whose achievements and strongly developed atomic and molecular physics at UMK gave an inspiration to create the National Laboratory of Atomic, Molecular and Optical Physics (KL FAMO) in Toruń:

a) I investigated the opto-magnetic double resonance phenomenon in ^{48}Cd atoms (analyzed problems: optical pumping, Zeeman effect, longitudinal and transverse relaxation times, Bloch equations).

b) I derived the analytical equation for the resonance curves (Majorana - Brossel) examination and made analysis of signals for the vaporized resonance quartz cells with cadmium ^{48}Cd.

c) I determined the lifetime of ^{48}Cd atoms in the excited state and the Lande factor.

Methodology of research and experimental setup were described in the publication-
S. Łęgowski, A. Molhem, G. Osiński, P. Rudecki, *Tensor polarizability of cadmium atoms in the excited state (5s5p)* $^3P_1$, The European Physical Journal D 35(2) (1995), and
Zeitschrift für Physik D Atoms, Molecules and Clusters **35**, 101-105 (1995).

- **Faculty of Physics, Warsaw University of Technology (2001-2011)**

The construction of EOS setup and investigation of photodetectors required to overcome many difficulties and through this demonstrate scientific activity (simultaneously with minimal financial support, except for the purchase of expensive femtosecond laser Spectra-Physics Tsunami HP). I cooperated with:

- Research Centre Jülich, Germany - dr M. Mikulics,
- Slovak University of Technology- prof. P. Kordoš,
- University of Rochester, USA - prof. dr hab. R. Sobolewski,
- National Centre for Nuclear Research in Świerk - dr J. Milczarek,
- Faculty of Chemistry at Warsaw University of Technology- prof. dr hab. K. Brudzewski (common manuscript),
- Institute of Electronic Technology in Warsaw,
- Institute of Electronic Materials Technology in Warsaw,
- Faculty of Mechatronics Warsaw University of Technology,

Cooperation encompassed: preparation of different elements for EOS system, investigation of the electro-optic properties of $LiTaO_3$ crystal, receiving photodetectors from Jülich and establishing cooperation, bounding devices and irradiation by neutrons, testing detector in Rochester (USA) and consultations.

- **Institute of Physics, Polish Academy of Sciences, Warsaw**

Cooperation:

- Korea University of Science and Technology, Center for Theoretical Physics of Complex Systems, Institute for Basic Science (IBS), Daejeon 34126, Korea, prof. Barbara Dietz – regular cooperation from 2016
- Center of Theoretical Physics of the Polish Academy of Sciences - prof. Adam Sawicki

International grants:

- **SHENG 1 -** "*Quantum and wave-dynamical Chaos in systems with integer or half-integer spin and preserved or violated time-reversal invariance: experiment and theory"* (06.2018-07.2023) - cooperation with prof. Barbara Dietz from Theoretical Physics, Lanzhou University, China - I was participant
- **EAgLE -** the European Action towards Leading Centre for Innovative Materials project- it supported the realization of manuscripts **[H1], [H2], [P17]**, **[P18].**

National grants:

1. **Opus 12 -** „*Influence of the topology of microwave networks simulating quantum graphs on their spectral and scattering properties and on the waveform of signals in the time domain"*(08.2017-07.2021) - participant
2. **Sonata 5** - "*Experimental study of spectral powers of discrete and finite series S(f) in microwave systems simulating chaotic quantum systems"* (04.2014-09.2017r.) - participant
3. **Miniatura 1 –** project no. 2017/01/X/ST2/00734 financed by the NCN- *"Experimental study of the properties of the elastic gain factor as a function of system openness"* (12.2017-11.2018) - **manager**

   Original project referred to one task, i.e. the study of enhancement factor in the GOE system with N-scattering channels. Nonetheless, I expanded this project by a new concept and decided to examine properties of system with partially broken time reversal symmetry in the presence of N-channels, taking into account incomplete spectra. I obtained interesting results which have been demonstrated in several publications: **[H3], [H4], [P5], [P7], [P13], [P15],** including three papers in Physical Review E.

4. **Opus 25** - I submitted the application *"Study of quantum systems with GSE symmetry in the RMT model"* – this project includes a new concept of GSE system belonging to the third fundamental symmetry class in RMT, taking into account the half-spin. Proposed model will incorporate two equally dispersing subgraphs **H** and **O** with mixed Neumann and Dirichlet boundary conditions.

5. Remaining achievements

Publications refer to the studies of different properties in low-dimensional systems.

[P1] A. Akhshani, **M. Białous,** L. Sirko
Quantum graphs and microwave networks as narrow-band filters with controllable transmission properties,
Phys. Rev. E - accepted (2023)

[P2] O. Farooq, A. Akhshani, **M. Białous**, Sz. Bauch, M. Ławniczak, L. Sirko
Investigation of the generalized Euler characteristic of graph and microwave networks split at edges and vertices,
Physica Scripta 024005 (2023)

[P3] M. Ławniczak, A. Akhshani, O. Farooq, **M. Białous**, Sz. Bauch, B. Dietz, L. Sirko
Distributions of the Wigner reaction matrix for microwave networks with symplectic symmetry in the presence of absorption,
Physical Review E **107**, 024203 (2023)

[P4] O. Farooq, A. Akhshani, **M. Białous**, Sz. Bauch, M. Ławniczak, L. Sirko
Experimental investigation of the generalized Euler characteristic of the networks split at edges,
Mathematics **10** 3587 (2022)

[P5] **M. Białous**, B. Dietz, L. Sirko
Power spectrum of discrete and finite series of levels in chaotic resonators with and without partially violated time - reversal symmetry. The case of missing levels.
Acta Physica Polonica A no. **6**, vol. 140, (2021)

[P6] M. Ławniczak, P. Kurasov, S. Bauch, **M. Białous,** A. Akhshani**,** L. Sirko
A new spectral invariant for quantum graphs,
Scientific Reports **11**, 15342 (2021)

[P7] **M. Białous**, B. Dietz, L. Sirko
Unusual properties of the enhancement factor in an open wave chaotic system with time-reversal - invariance violation,
Acta Physica Polonica A No.**4** Vol.139 (2021)

[P8] M. Ławniczak, P. Kurasov, S. Bauch, **M. Białous,** L. Sirko
Euler characteristic of graphs and networks,
Acta Physica Polonica A No.**3** Vol.139 (2021)

[P9] M. Ławniczak, J. Lipovsky, **M. Białous**, L. Sirko
Application of topological resonances in experimental investigation of a Fermi golden rule in microwave networks,
Physical Review E **103** 032208 (2021)

[P10] M. Ławniczak, A. Sawicki, **M. Białous,** L. Sirko
Isoscattering strings of concatenating graphs and networks,
Scientific Reports **11**, 1575 (2021)

[P11] M. Ławniczak, P. Kurasov, S. Bauch, **M. Białous**, V. Yunko, L, Sirko
Hearing Euler characteristic of graphs,
Physical Review E **101**, 052320 (2020)

[P12] **M. Białous**, P. Dulian, A. Sawicki, L. Sirko
Delay - time distribution in the scattering of short Gaussian pulses in microwave networks,
Physical Review E **104,** 024223 (2021)

[P13] **M. Białous**, B. Dietz, L. Sirko
Investigation of the elastic enhancement factor in microwave chaotic cavities in the presence of strong opened channels,
Acta Physica Polonica A, no **5** vol 136, 16215 (2019)

[P14] M. Ławniczak, Sz. Bauch, V. Yunko, **M. Białous**, J. Wrochna, L. Sirko
Investigation of the Wigner's reaction matrix of microwave networks simulating quantum graphs with broken time reversal symmetry -one-port investigation,
Acta Physica Polonica A, no 5 vol **136**, 16209 (2019)

[P15] **M. Białous**, B. Dietz, L. Sirko
Experimental investigation of the elastic enhancement factor in a microwave cavity emulating a chaotic scattering system with varying openness,
Physical Review E **100**, 012210 (2019)

[P16] M. Ławniczak, **M. Białous**, V.Yunko, Sz. Bauch, L. Sirko
Missing level statistics and power spectrum analysis of three -dimensional chaotic microwave cavities,
Physical Review E **98** 012206 (2018)

[P17] M. Ławniczak, **M. Białous**, V.Yunko, Sz. Bauch, L. Sirko
Analysis of missing level statistics for microwave networks simulating quantum chaotic graphs without time reversal symmetry-the case of randomly lost resonances,
Acta Physica Polonica A no **6** vol.132 (2017)

[P18] **M. Białous**, V. Yunko, Sz. Bauch, M. Ławniczak, B. Dietz, L. Sirko
Long-range correlations in rectangular cavities containing pointlike perturbations,
Physical Review E **94**, 042210 (2016)

[P19] M. Ławniczak, **M. Białous**, V.Yunko, Sz. Bauch, L. Sirko
Numerical and experimental studies of the elastic enhancement factor for 2D open systems,
Acta Physica Polonica A no 6 vol.**128**, 974 (2015)

[P20] **M. Białous**, L. Sirko, V. Yunko, Sz. Bauch, M. Ławniczak
Investigation of the enhancement factor in a transient region between regular and chaotic dynamics,
1st URSI Atlantic Radio Science Conference, URSI AT-RASC, IEEE (2015)

[P21] M. Ławniczak, **M. Białous**, V. Yunko, Sz. Bauch, L. Sirko
Experimental investigation of the enhancement factor in a transient region between regular and chaotic dynamics,
Physical Review E **91**, 032925 (2015)

- Post - conference reviewed papers / chapters:

[K1] M. Ławniczak, O. Farooq, A. Akhshani, **M. Białous**, Sz. Bauch, L. Sirko
Role of the boundary conditions in the graphs split at vertices,
Proceedings of the 15th Chaotic Modeling and Simulation International Conference, 14-17 June 2022, Athens, Greece, Editor Christos H. Skiadas, Springer Proceedings in Complexity, p.165-175 (2023)

[K2] M. Ławniczak, P. Kurasov, Sz. Bauch, **M. Białous**, L. Sirko
The relationship between the Euler characteristic and the spectra of graphs and networks,
13th Chaotic Modeling and Simulation International Conference, Editor Christos H. Skiadas, Florence, Italy, 9-12 June 2021, p. 487-497, Springer (2021)

[K3] M. Ławniczak, A. Sawicki, **M. Białous,** L. Sirko
Isoscattering chains of graphs and networks,
14th Chaotic Modeling and Simulation International Conference, Chaos, Athens, Greece (2021)

[K4] M. Ławniczak, **M. Białous**, V. Yunko, Sz. Bauch, L, Sirko
Missing- level statistics in chaotic microwave networks versus level statistics of partially chaotic systems,
Discrete and Continuous Models in the Theory of Networks ,vol. 281, p. 241-253, Springer (2020)

[K5] L. Sirko, **M. Białous**, S. Bauch, P. Kurasov, J. Lipovsky, B. Dietz, M. Ławniczak
Experimental investigations of the Euler characteristic and other peculiar properties of microwave networks and graphs,
Quantum graphs in Mathematics, Physics and Applications, Stockholm University 8-9 December (2020)

[K6] V. Yunko, **M. Białous**, S. Bauch, M. Ławniczak, L. Sirko
Experimental and numerical studies of spectral properties of three-dimensional chaotic microwave cavities: The case of missing levels,

11th Chaotic Modeling and Simulation International Conference, p. 303-315, Springer Proceedings in Complexity, Springer (2019)

[K7] M. Ławniczak, **M. Białous**, V. Yunko, Sz. Bauch, B. Dietz, L. Sirko
Influence of Topology and Absorption on Properties of Quantum Graphs and Microwave Networks,
1th Chaotic Modeling and Simulation International Conf. Rome, Italy 5-8 June (2018).

Małgorzata Białous
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(applicant's signature)